Modeling fertility by order of birth

Antoine Pierrard, Giuliana Zegarra Beltran & Ester Rizzi
Université Catholique de Louvain

Introduction

Although the modeling of age specific fertility schedules has been exposed in a lot of researches (de Beer, 2011), modeling fertility by birth order has received little attention. Several works have attempted to fit observed parity specific schedules (Zhu, 1994), but none of them expands to models that could be used to estimate parity specific schedules when only data on all births combined fertility are available.

We propose to derive parity specific fertility rates from an overall age-specific (all births combined). Based on model proposed by Peristera and Kostaki (2007) for fitting fertility curves, we explore the assumption that there is a relationship between all the parameters of their model for all births combined and for parity specific, and derive several sets of coefficients to estimate age and parity specific fertility schedules from age and all birth combined fertility schedules.

1. Data

The data used in the estimation and validation of the model coefficients come from the Human Fertility Database. For now, all the countries for which age and parity specific fertility schedules are available have been taken into account. It should be noted that they do not systematically end up in the analysis, since the algorithm used to fit the Peristera & Kostaki model sometimes fails to converge toward a single solution.

2. Method

We rely on the model proposed by Peristera & Kostaki (2007):

$$f_x = c_1 \exp\left(-\left(\frac{x-\mu_1}{\sigma_1(x)}\right)^2\right) + c_2 \exp\left(-\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right)$$

where $\sigma_1(x) = \sigma_{11}$ if $x \leq \mu$ and $\sigma_1(x) = \sigma_{12}$ if $x > \mu$

The hypothesis underlying our model is as follow: all the parameters of a Peristera & Kostaki fit are good predictors of parameters for birth order fertility schedules. In other words, given a single all births combined fertility schedule, it can be derived a set of corresponding age and parity specific

fertility rates. Our purpose is then to provide tables of coefficients to estimate the parameters of a Peristera & Kostaki function for any specified birth order fertility rates.

First, all birth combined fertility schedules are fit using a Peristera and Kostaki function, which returns a set of parameters. The same is done for parity specific schedules. Second, linear regressions are applied, with one parity specific parameters as the dependent variable, and all birth combined parameters as predictors.

This gives the series of regression coefficients that can be used to estimate any parity parameters when only all births parameters are available. For now, three sets of model coefficients have been defined based on the TFR (all orders combined) of the observed schedule: high (TFR > 2.1), moderate and low (TFR < 1.3).

3. Results : estimating age and parity specific fertility schedules

Provided with any age specific fertility schedule, all birth combined, the procedure to compute schedules for first, second and third birth orders is as follow, with the schedule observed for Austria in 1990 as an example:

Step 1. Fit the age-specific fertility schedule (all births combined) with a Peristera & Kostaki function. This gives the first set of parameters:

μ_1	μ_2	σ_{11}	σ_{12}	σ_2	c_1	c_2
25.81	21.23	3.16	8.58	3.56	0.09	0.06

Step 2. Given the TFR of the original schedule, identify the appropriate set of model coefficients, for the chosen birth order. The TFR corresponding to the Austrian schedule in 1990 being 1.46, the coefficients are taken from the Moderate Fertility table (see Appendix for details).

Step 3. Compute the parameters of the parity schedule to be estimated, as a linear function of the parameters (step 1) and the coefficients (step 2).

For the first parameter (μ_1^*), the equation would be:

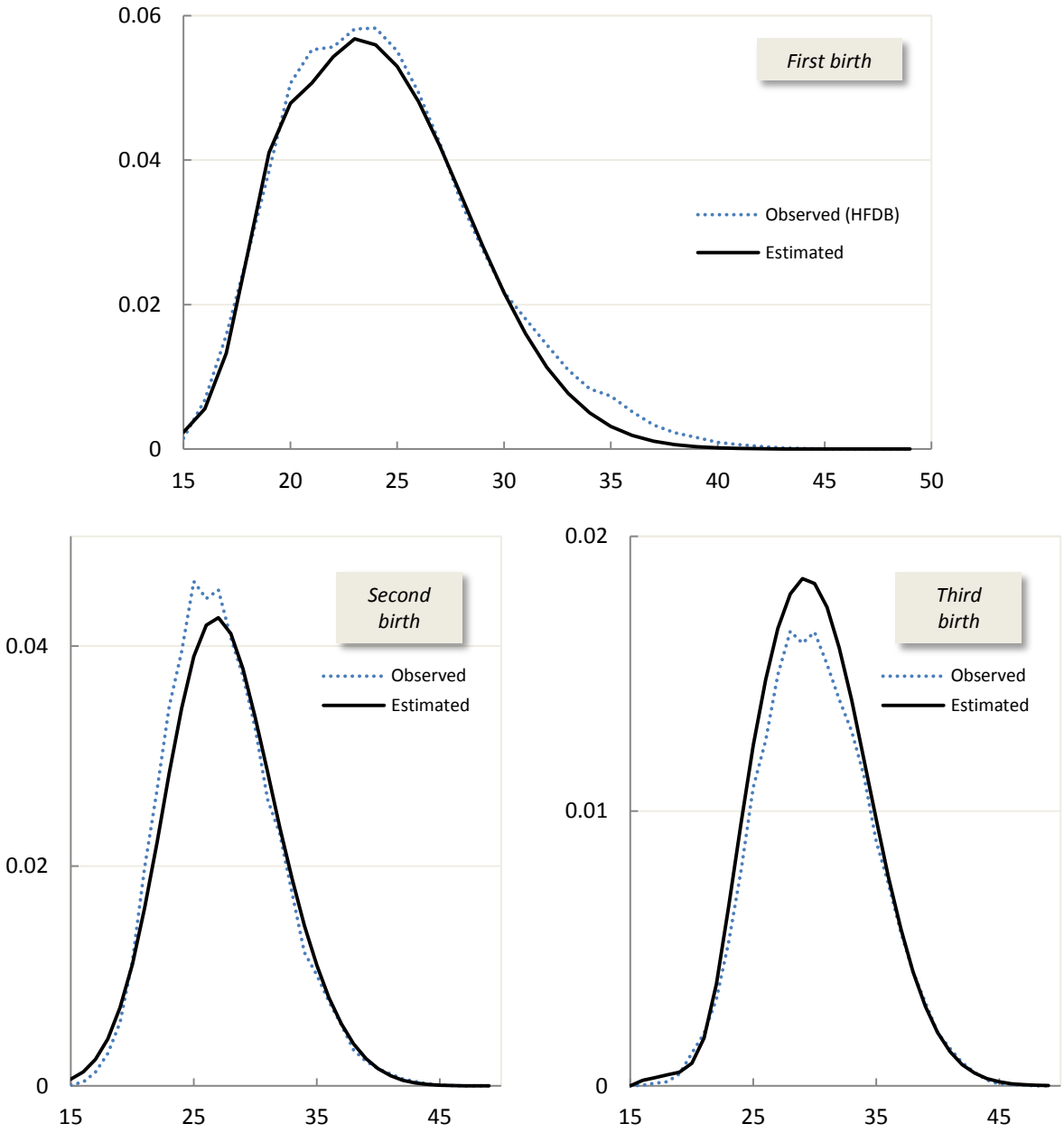
$$\mu_1^* = \mathrm{a} + \mathrm{b} * \mu_1 + \mathrm{c} * \mu_2 + \mathrm{d} * \sigma_{11} + \mathrm{e} * \sigma_{12} + \mathrm{f} * \sigma_2 + \mathrm{g} * c_1 + \mathrm{h} * c_2$$

Repeated for all parameters, this gives:

μ_1^*	μ_2^*	σ_{11}^*	σ_{12}^*	σ_2^*	c_1^*	c_2^*
23.18	19.10	4.55	6.94	1.80	0.05	0.01

Step 4. Compute the set of age and parity specific fertility rates using the Peristera & Kostaki formula and appropriate parameters from step 3.

The figures below compare the estimated fertility rates (for order 1, 2 and 3) with the ones available in the Human Fertility Database.



Conclusion: expected improvements

Although our results show encouraging perspectives, we expect to improve the fit of the model. As mentioned above, the parameters of the Peristera & Kostaki model are computed using non-linear estimation. The “nls” function (from the R statistical software) often fails to converge to a solution, which diminishes the amount of fertility schedules available. We mainly plan to propose different sets of coefficients that provide better estimation of fertility by birth order.

References

de Beer, J. (2011). A new relational method for smoothing and projecting age-specific fertility rates: TOPALS. *Demographic Research*, 24, 409-454.

Human Fertility Database. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at www.humanfertility.org (data downloaded on March 3rd 2012)

Peristera, P., & Kostaki, A. (2007). Modeling Fertility in Modern Populations. *Demographic Research, 16*, 141-194.

Zhu, J. (1994). A model of the age patterns of births by parity in natural fertility populations. *Mathematical Population Studies, 4*(3), 153-173.

Appendix

Model parameters tables for moderate fertility levels

First birth

	a	b	c	d	e	f	g	h
μ_1	1.791	1.538	-0.500	-1.039	-0.145	-0.210	3.254	-40.753
μ_2	0.079	0.631	0.366	-0.460	0.106	-1.131	3.212	-11.385
σ_{11}	15.643	-0.083	-0.280	-0.318	-0.600	1.050	9.757	-23.139
σ_{12}	-10.068	-0.050	0.526	0.057	0.741	-0.090	10.005	-1.301
σ_2	-3.359	0.380	-0.105	-0.003	-0.060	-0.296	-8.381	-0.276
c_1	0.157	-0.003	-0.004	0.008	-0.005	0.004	0.310	0.396
c_2	-0.097	0.005	-0.002	0.001	0.006	-0.007	-0.086	0.171

Second birth

	a	b	c	d	e	f	g	h
μ_1	26.778	0.577	-0.939	-0.175	0.415	1.058	-0.718	-37.090
μ_2	-1.917	0.815	0.383	-0.158	1.135	-0.708	3.560	-50.653
σ_{11}	-0.066	0.641	-0.755	-0.381	0.614	1.000	-5.027	-32.095
σ_{12}	2.330	-0.017	0.172	-0.117	0.309	-0.414	-11.666	4.962
σ_2	4.617	-0.155	0.202	0.099	0.064	-0.068	-2.727	13.122
c_1	-0.012	-0.003	0.006	0.005	0.001	-0.007	-0.012	0.145
c_2	0.143	0.001	-0.007	-0.001	-0.003	0.008	0.001	0.127

Third birth

	a	b	c	d	e	f	g	h
μ_1	14.410	1.162	-0.601	-0.560	-0.154	0.210	13.910	-21.371
μ_2	18.712	-0.392	0.554	0.570	0.206	0.186	-20.584	-21.139
σ_{11}	-43.722	1.108	0.440	-0.307	2.312	-1.804	-4.429	11.195
σ_{12}	5.880	-0.331	0.275	0.178	0.425	-0.002	-5.536	5.098
σ_2	-10.419	-0.070	1.574	-1.632	-0.248	-1.720	5.455	-83.944
c_1	-0.089	-0.001	0.003	0.003	0.005	-0.002	0.148	0.117
c_2	0.099	-0.001	-0.003	0.001	-0.005	0.006	0.006	-0.052