Comparative importance of the fertility model, the total fertility, the mean age and the standard deviation of age at childbearing in population projections

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Abstract

Using empirical fertility rates and population distributions, we study comparative contributions to births' prediction errors of choices for the fertility model and of the approximation errors of three main fertility indicators (the total fertility, the mean and the standard deviation of age at birth, respectively: TFR, MAB, SDAB). Agreeing with theories of dynamic populations, we find high importance of accuracy of TFR and MAB. Yet, the role is limited in population projections of the estimates of SDAB and of the choice of the fertility model form. More attention may be paid in population projections to working out (interdependent) scenarios for TFR and MAB, while relaxing complexity of other aspects of fertility projection models. Our results suggest widening the uncertainty range for TFR in cases when the MAB projections are based on regressions on TFR or other simplified assumptions.

1. Introduction. General considerations and motivation

The period total fertility rate (TFR) is the prime, if not the only, fertility indicator commonly used in producing the population projection scenarios (Lutz & Scherbov 2004; Lutz et al.

2004; Alders & de Beer 2004, Alders et al. 2007; U.S. Census Bureau. 2008; Heilig et al. 2010; Alkema et al. 2011; United Nations 2011; Eurostat 2011). The age distribution of fertility, usually, remains behind the scene and is rarely reported in the projection literature. In some applications, the age pattern of fertility is obtained by scaling up or down the baseline profile according the projected TFR levels; in others, the age pattern of fertility is linked, e.g., through regressions or model fertility curves, to the prime index, the TFR. Yet in other applications, the level (TFR) and age structure of fertility are forecasted by independent time series although theory suggests the two may be linked in processes of fertility postponement and anticipation.

Such simplifications may work well for low-mortality, no-migration stationary populations where population sizes of fertile age groups are equal and any age profile of fertility rates produces the same number of births as long as TFR is fixed. In a population with *n* persons in each of the single-year fertile age groups, there will be TFR**n* births each year. More realistically, population sizes of fertile age groups may vary substantially because of time-varying sizes of birth cohorts and effects of migration and mortality. Such irregular age profiles may produce differing numbers of births and projected populations when combined with fertility curves of the same total fertility but differing age patterns.

Commonly, the age pattern of fertility rates is captured by the mean and the standard deviation of age at childbearing (MAB and SDAB, respectively) – indicators found important in fertility and population models. Some models of age-specific fertility involve three parameters, which may be linked to the TFR, MAB and SDAB (Mitra 1967; Mitra & Romaniuk 1973; Romaniuk 1973; Brass 1974, 1975, 1978; Booth 1984; Pollard & Valkovics 1992; reduced models in Congdon 1993; Yi et al. 2000). Theories explaining the period fertility in terms of the share number and timing of births also point to the same set of indicators (theories by Ryder, 1951, and Bongaarts-Feeny, 1998, involve TFR and MAB; Kohler & Philipov, 2001, add the variance of age at birth). This accords to the models of dynamic populations which also point to importance of MAB (but less so – of other aspects of fertility age profile) in long-term population change (Keyfitz 1971; Kim et al. 1991; Ediev 2005, 2011). More complicated fertility models involving more than three parameters have also been proposed in the literature and shown to better fit the empirical fertility schedules (Coale & Trussell 1974; Hoem et al. 1981; Thompson et al. 1989; Congdon 1993, Chandola et al. 1999; Schmertmann 2003; Peristera & Kostaki 2007).

Despite extensive literature on modeling and fitting the fertility curves, questions remain open of weather complicating the fertility model may improve projections or, more instructively, what is the relative contribution of model choice and parameterization into accuracy of projected births. With respect to alternative methods of fitting the parameters in the special case of Pearsonian Type I fertility curve, contributions to the births prediction errors were studied by Mitra and Romaniuk (1973). In their study, all alternative parameterization methods produced only minor prediction errors (fractions of the percent for the most of the years presented) on Canadian data. While this was an important hint to the possible role of the model choice in fertility projections, Mitra and Romaniuk did not cover models other than the Pearsonian Type I and did not consider parameterizations with imperfect values of the MAB.

Our work aims to broaden understanding of the role of model choice and parameterization in fertility projections through a comparative analysis of importance of all three main fertility indicators in projecting the number of births. Model-wise, we consider several alternative fertility models of different sophistication levels. Using empirical population compositions and fertility rates, we study deviations of the predicted number of births from the empirical number under alternative models of the age pattern of fertility rates and for different approximations of the major fertility indicators. The recently launched and expanding Human Fertility Database (2012), HFD, provides vast empirical data base for our study.

In the next section, we present data, fertility models and prediction error indicators used in the study. Then, we present results for the prediction errors and conclude by discussion.

2. Data and methods

Appreciating the sensitivity of the projected number of births to accuracy of the TFR is straightforward. A one percent error in TFR, the age pattern of fertility being intact, would result in a similar one percent error in the projected number of births. Sensitivity to the deficiencies in the projected age pattern of fertility is, however, harder to study. Impact of such deficiencies depends not only on the extent of those deficiencies but also on the age composition of the population to which the fertility pattern is applied. In a population with a 'flat' age pattern at fertile ages the accuracy of the fertility pattern (given TFR is fixed) hardly matters. On the contrary, in projecting a population with strongly skewed age composition applying the same TFR to younger or older cohorts at childbearing ages may produce different numbers of births.

To explore this effect, we use the Human Fertility Database (2012), HFD, and simulate percentage errors of the predicted numbers of births assuming variety of typical model simplifications about the age pattern of fertility rates. As typical in projections, we only consider age aspect of fertility and not the parity or cohort aspects of it. Therefore, we use only the age profiles of period fertility rates and female population age compositions from the HFD. The database contains estimates for 1483 populations, some overlapping with each other, over long time period (starting in 1891 for Sweden but later for other populations).

We apply to the empirical female population a model age profile of fertility rates, with TFR set at the empirical level, and see how far is deviating the imputed number of births from the empirical number. To shed the light on the role of the form of the fertility model, we use the following model age patterns of fertility rates, $f(x)$ (F_x in discrete form): direct transformation of the empirical schedule; two variants of the Brass model; Schmertmann's Quadratic-Spline model; the Gamma model; the Rectangular model; and the Ryderian pentapartite model. The models are described next.

Model 1. Direct transformation of the empirical fertility schedule.

First, we transform the empirical pattern $f^{e}(x)$ of fertility rates directly, according to the assumed model values of the MAB and SDAB:

$$f(x) = \frac{1}{k} f^{e}\left(MAB + \frac{x - MAB^{e}}{k}\right), \tag{1}$$

here superscript 'e' notes the empirical schedule and $k = \dfrac{SDAB}{SDAB^{e}}$. Multiplier '$\dfrac{1}{k}$' before the empirical schedule assures identical TFR's in the empirical and transformed schedules; in calculations, we use discrete approximation to (1) and slightly adjust the multiplier to match exactly the assumed TFR. Occasionally, the transformed rates may turn non-plausible when positive at ages beyond the fertile age limits. We overcome this problem by applying (1) only in the fertile age range and setting fertility rates zero outside that range.

Indeed, there would be no known empirical schedule of future fertility rates to which to apply the transformation above in real-life population projections. Only in the short-range projections, when projected TFR, MAB and SDAB do not differ much from their baseline values, could one use the baseline profile as a basis for the transformation. Our next model addresses this problem.

Model 2. Transformation of the regression-based fertility schedule.

In this model, we emulate the realistic case when the empirical schedule is, in fact, unknown to the forecaster. Therefore, we approximate the baseline age pattern in (1) through a linear regression of age-specific fertility rates on the TFR and MAB:

$$F_x \approx a_x + b_x TFR + c_x MAB\,, \tag{2}$$

where the regression coefficients are estimated over the entire database. Adding SDAB as a predicting variable in (2) does not improve noticeably the prediction efficiency of the regression. At the same time, dropping MAB from (2) reduces the fit (R^2 averaged over all fertile ages drops from 65% to 39%). Once the baseline pattern (2) is set, we apply transformation (1) to fit the assumed values for TFR, MAB, and SDAB.

Model 3. The Brass relational model with the empirical fertility schedule as the standard.

The Brass relational model is a convenient method for transforming the baseline fertility pattern into a new (and plausible) set of fertility rates:

$$F_x = TFR \cdot \left(\Phi_{x+1} - \Phi_x\right), \tag{3}$$

where $\Phi_x = \frac{\sum_{x=0}^{x-1} F_y}{TFR}$, $\Phi_0 = 0$, is the normalized cumulative fertility related to the standard normalized cumulative fertility $\Phi_x^{\,*}$:

$$\Phi_x = \frac{1}{1 + \exp\left(-\alpha - \beta \ln\left(\frac{\Phi_x^{\,*}}{1-\Phi_x^{\,*}}\right)\right)},\ x > 0, \tag{4}$$

parameters α and β determine MAB and SDAB.

In our first version of the model, the empirical fertility schedule is used as the standard:

$$\Phi_x^{\,*} = \frac{\sum_{x=0}^{x-1} F_y^{\,e}}{TFR^e},\ \Phi_0^{\,*} = 0, \tag{5}$$

and use optimization procedure to find model parameters α and β producing the assumed MAB and SDAB.

Model 4. The Brass relational model with the regression-based standard.

Our second variant of the Brass model is similar to the first variant, except the standard fertility profile is produced through linear regressions (2) and not taken directly from the empirical profile.

Model 5. Schmertmann's Quadratic-Spline model, QS.

Parametric fertility models were found useful in many applications but also shown to fail capturing some aspects of empirical fertility. To see, how big might be the errors because of the parametric form of the model as well as due to deviations of the model parameters form their empirical values, we use four models: two sophisticated (Schmertmann's (2003) Quadratic-Spline model, QS, and the following Gamma model) and two simplistic (Rectangular and Ryderian pentapartite models, come next after the Gamma model) ones.

Details of the QS model may be found in (Schmertmann 2003). In brief, the model assumes three special points on the age scale: the youngest age at which fertility rises above zero (α), the age at maximum fertility (P) and the youngest age above P at which fertility falls to half of its peak (H). The youngest age at positive fertility (α) is set, as in (Schmertmann 2003), to constant level $\alpha = 15$ in all our calculations. Hence, the model, effectively, is based on two parameters, P and H. the full age schedule of fertility rates is derived by fitting quadratic polynomials in between age 'knots', the knots being functions of the model parameters. The readers may refer to (Schmertmann 2003) for detailed calculation formulas. In our study, we use non-matrix calculation formulas (Appendix B in Schmertmann 2003). Unlike in the original method, we use optimization procedures to find parameters P and H in order to fit exactly the assumed values for MAB and SDAB of the fertility schedule, and not to fit the entire schedule of the age-specific fertility rates. Once the schedule is found that matches assumed MAB and SDAB, we scale the schedule up or down in order fit exactly the assumed level of the TFR.

<u>*Model 6. The Gamma model.*</u>

In the Gamma model,

$$F_x = \frac{1}{\Gamma(\alpha_3)} \alpha_1 \alpha_2^{\alpha_3} (x - \alpha_4)^{\alpha_3 - 1} \exp(-\alpha_2 (x - \alpha_4)), \; x \geq \alpha_4 , \tag{6}$$

where α_i, $i = \overline{1,4}$, are model parameters and $\Gamma(p) = \int_0^\infty u^{p-1} \exp(-u) du$. This model is convenient for our and other applications in possibility to derive the model parameters from substantive demographic considerations. Unlike in other works, where the parameters were selected to fit the whole age pattern of fertility rates, we select the parameters so that to reproduce exactly the key fertility indicators (TFR, MAB, and SDAB). To this end, we use the following relations:

$$\alpha_1 = TFR \tag{7}$$

(in practice, there is slight upward adjustment to this parameter, to compensate for two missing tails of the Gamma curve that fall beyond reproductive ages),

$$\alpha_4 + \frac{\alpha_3}{\alpha_2} = MAB , \tag{8}$$

$$\sqrt{\frac{\alpha_3}{\alpha_2^{\ 2}}} = SDAB . \tag{9}$$

Relations (7)-(9) impose three restrictions on four parameters of the model. Given, for example, the value of the minimum age α_4 of the Gamma curve, all other parameters may be derived analytically. Therefore, the task of fitting the model to the assumed fertility schedule is reduced to choosing a single parameter α_4, for which we use a simplified approximation

$$\alpha_4 \equiv 0 . \tag{10}$$

This assumption is supported by research showing the parameter turns zero after around 1980s in low-fertility countries (Thompson et al. 1989, Keilman & Pham 2000). Although, this might not be the best solution in higher-fertility contexts, our study shows the choice for

α_4 is of low importance in population projections when compared with the impact of the choice for other parameters. Given (10), one can use closed-form analytical expressions for the other parameters of the model:

$$\alpha_2 = \frac{MAB}{SDAB^2}, \tag{11}$$

$$\alpha_3 = \left(\frac{MAB}{SDAB}\right)^2. \tag{12}$$

These relations allow deriving the model age structure of fertility rates from the projected TFR, MAB, and SDAB.

<u>*Model 7. The Rectangular model.*</u>

The Rectangular, as well as the following Ryder's, model serve as examples of an 'extreme-crude' model of the age pattern of fertility rates. It is based on assuming age-independent fertility in age range $x \in [a,b]$ and zero fertility outside the range:

$$f(x) = \begin{cases} \frac{TFR}{b-a}, x \in [a,b], \\ 0, x \notin [a,b], \end{cases} \tag{13}$$

where $a = MAB - \sqrt{3}SDAB$ and $b = MAB + \sqrt{3}SDAB$ are set to match the assumed MAB and SDAB.

<u>*Model 8. Ryder's (pentapartite) model.*</u>

Studying possibility to forecast births using cohort-period translation of fertility rates, Ryder (1989) proposed a 'tetrapartite' model, where fertility rates are all set zero except at four equally-distanced ages (Ryder suggests, in particular, to use ages 17.5, 24.5, 31.5, and 38.5 for that purpose). As rough as it is, the model showed good results in translating the period TFR and MAB from first three moments of the cohort fertility function. Our calculations, however, showed that the model produces high prediction errors, when compared with other alternatives; better results are produced when increasing the number of ages with non-zero fertility. Here, we present results for the pentapartite model where fertility rates are all zero except at five equally spaced ages 17.5 to 37.5. This model, having errors (ca. 1.88 percent in the number of births) moderately higher than the best Ryderian alternative (an octopartite model with eight parameters and ca. 1.38 percent errors) but lower than the Ryder's tetrapartite (errors ca. 2.62 percent), provides a good tradeoff between the prediction errors and the number of parameters. Another difference of our work to the original Ryder's approach is that we do not consider moments of the age distribution of fertility rates except for the first two (corresponding to MAB and SDAB, respectively), because of small contribution of higher-order moments into the prediction accuracy. So, our Ryderian model assumes non-zero fertility only at ages 17.5, 22.5, 27.5, 32.5, and 37.5 (mid-points of corresponding single-year-long age intervals). Fertility rates at those ages are selected to fit the assumed values for TFR, MAB, SDAB, and, to improve the robustness of the procedure, to minimize the sum of squares of the rates.

As well described in the literature (see the introduction), the seven models used here do not exhaust the options for the models of the fertility curve. Yet, our choice covers wide range of model complexity and includes some commonly used and convenient models. Usage of the method of moments for parameterizing the fertility models is simpler than the usual fitting to the empirical age-specific rates (assuming those rates are known). Its ability to reproduce exactly the assumed TFR, MAB, and SDAB is convenient in projections (when there is no, in fact, a known empirical schedule to which to fit the model) and in our study of errors arising from deviation of model fertility parameters from their true values.

Indicators of the prediction error

Our indicator of prediction accuracy is the mean squared relative error (MSRE) of the predicted number of births:

$$MSRE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\hat{B}_i - B_i}{B_i}\right)^2}\ , \tag{14}$$

where B_i and $\hat{B}_i$ are the benchmark and predicted numbers of births obtained by applying the exact and the approximate fertility rates, respectively, to population '*i*'; $N = 1483$ is the total number of populations in the *HFD*. This indicator in convenient in possibility to compare it, one-to-one, to the errors induced by the biases in the TFR (a one per cent bias in TFR produces a similar error in the projected number births).

When the approximate schedule comes from (relational) models one through three, the benchmark B_i is simply the empirical number of births from *HFD* in respective population.

For all other (parametric) models we use two benchmark numbers of births: the empirical one and the one generated by the same model when TFR, MAB and SDAB are set at their empirical values. The first benchmark yields errors combining contributions from limitations of both the model form and model parameterization. The second benchmark yields errors associated only with the choice of the fertility parameters and not with the inability of the model to reproduce peculiarities of the empirical fertility pattern.

The models' own errors are estimated by comparing the empirical numbers of births to those produced by the models under true values for TFR, MAB, and SDAB (in this definition, models 1 and 3 have null own errors). Note that own errors of models (except models 1 and 3) also include errors due to neglect of the higher-than-two-order moments of the fertility age pattern.

Models' parameterization

In our calculations, the TFR is always set at its empirical level, but MAB and SDAB vary according the following choices:

- the exact empirical value from the HFD;
- the average over all HFD populations (27.7 and 5.56 years for the MAB and SDAB, respectively);
- country-specific average, over HFD data for relevant population only;
- linear regression-based approximation with the TFR used as the predictor (regressions are fit over the whole HFD):

  $MAB = 26.4 + 0.63 \cdot TFR$, R^2=6 percent,

  $SDAB = 4.42 + 0.55 \cdot TFR$, R^2=44 percent;
- country-specific linear regressions with the TFR used as the predictor (R^2 averaged over all HFD populations is 15 percent for MAB and 43 percent for the SDAB; those statistics vary substantially form population to population).

3. Results

Mean squared relative errors

MSRE's estimated for different models and parameterizations are presented in Table 1. For a reference, the table also features the mean squared annual change of TFR (4.23 percent), which could be useful to compare the MSRE's to.

Table 1 around here.

All selected fertility models, even the cruder Rectangular and Ryderian models, produce similar errors at similar choices for the fertility parameters when the models are assessed vs. their own best parameterizations. This suggests that errors induced by approximations to the main fertility parameters may be nearly independent of the fertility model form.

When compared to the empirical numbers of births models' own errors also contribute to MSREs. The Rectangular model and the Ryderian pentapartite, in particular, perform considerably worse than other models. Models' own errors (last but one row of the table) mount to 0.42 percent (regressions-based fertility profiles of Model 2), 0.51 percent (the Brass model with regressions-based standard), 0.57 percent (Schmertmann's QS model), 0.72 percent (the Gamma model), 1.56 percent (the Rectangular model), and 1.88 percent (the Ryderian pentapartite model).

When compared to these natural reference levels, all models perform well when the true MAB is known but SDAB is approximated (in other words, uncertainty in SDAB does not substantially worsen the prediction errors as compared to the models' own errors). Even substituting the true SDAB by the average over the entire HFD produces good predictions: the corresponding MSRE's differ substantially from the models' own errors only for the models with low errors (the relational models and the QS). Even better estimate for SDAB is the one based on country-specific regression on TFR (adding MAB as an explanatory variable slightly improves the results; yet, we do not consider that model here). Given true MAB values, prediction errors because of both the model structure (except for the Rectangular and Ryderian models) and parameterization deficiencies are of a smaller order of magnitude than the annual variation in TFR. Therefore, model choice or extra knowledge about SDAB would hardly matter in projection exercises with uncertain TFR.

Imperfect estimates of MAB, including the regressions on TFR, produce errors exceeding the models' own errors by more than twice (except for the Rectangular and Ryderian models with large own errors) and of the size comparable to the uncertainty in TFR. Because of weak association between the MAB and TFR, even on a country-specific basis, the regression-based MAB's produce results not much better than the averages-based approximations. As yet another reference for the first five models, one may use the projection errors of the cruder Rectangular and Ryderian model (1.56 and 1.88 percent, respectively): lack of knowledge about MAB produces nearly as high errors as those produced by reduction of the fertility age pattern into a simple rectangular shape or a set of five ages with non-zero fertility.

Once the knowledge about MAB is lost, accuracy of SDAB does not matter anymore (according to our rough assessments above, MSRE's for similarly approximated MAB's but different assumptions for the SDAB are not significantly different). Projections based on estimated MAB but true values for SDAB are occasionally even worse than the projections based on the estimated values for both parameters. This puzzling result may indicate that once MAB deviates from its true value, use of true SDAB's may worsen the accuracy of the projection by introducing extra source of fluctuation of the outcome compared to using more stable estimated SDAB's.

When choosing between the models, any one of them, except for the Rectangular and the Ryderian models, produce acceptable results, the simplest transformation models being, surprisingly, the best ones. Also notably, the Schmertmann's QS model, with the same number of parameters but somewhat more stylized age pattern, is substantially better-off than the Gamma model. In births prediction accuracy, the QS model is closer to the relational models.

Our observations are supported by distributions of prediction errors of the fertility models under alternative parameterizations (see histograms of relative percentage errors, with the empirical births used as benchmarks, for models except for Models 1 and 3, in Figure 1). As seen from the histograms, lack of knowledge about SDAB and even the model choice (except for choosing the Rectangular and Ryderian models) hardly matter for the prediction accuracy, while better MAB's do make a difference. Fertility rates estimated by transforming the baseline fertility schedule (obtained, in its turn, from regressions on TFR) perform better both in MSRE's and as having less pronounced tails of the errors' distribution. When compared to the respective normal distributions, errors of most of the models show large excess kurtoses (shown on the plots). This may be explained, in part, by a combination of two effects. First, prediction errors are low, at any choice for the MAB and SDAB, for populations with relatively 'flat' (age-independent) age composition at fertile ages. These populations contribute to the peak around zero value in the errors' distribution (a closer examination also shows that errors decline even faster when the population gets closer to a 'flat' age composition at fertile ages). Second, the prediction errors decline faster when the model parameters get closer to their empirical values (we observe that this is true when MAB gets closer to its empirical value; but the same is not true for SDAB). Both these causes increase the number of almost-the-perfect-fit cases at the expense of only-a-good-fit ones.

Figure 1 around here.

Autocorrelations

For projections that are usually done for a set of consecutive years, it is important not only to know the magnitude of the errors in predicting the number of births in a single year, but also to know, if the errors cumulate over time or, rather, compensate each other if random and uncorrelated. Since HFD contains, for each population, data for rather long time series, we can study the autocorrelations between prediction errors in years t and t+1 empirically. Empirical autocorrelations estimated for each HFD population and averaged over all populations are presented in Table 2.

Errors induced by biases in MAB are not only higher in their magnitude; they are also tighter correlated for adjacent projection years. In other words, projection errors due to mistaken values of MAB cumulate stronger than the errors because of mistakes in SDAB or limitations in the model structure. In table 2, we also present autocorrelations for lag of two years. These autocorrelations also show that errors due to unknown MAB cumulate stronger than those due to unknown SDAB or limitations of the model structure. Autocorrelations at lag of two years are substantially lower than the squared autocorrelations for the lag of one year and indicate that errors cumulate slower than in an AR(1) process.

Substantially lower autocorrelations for errors of the Ryderian model may be explained by pronounced short-term variation of the errors due to the concentration of model fertility into five selected ages. Such a concentration augments impact of short-term age-to-age variation in the population size. It does not, however, lead to lower accumulation of errors over a long time, as we see next.

Table 2 around here.

Projection errors accumulated over one generation

Our next indicator concerns the prediction errors for births cumulated over time horizon of about one generation length (which we take for 27 years), a characteristic time block in population projections. We calculate this indicator as the mean squared value of relative prediction errors averaged over 27 years; all country-specific 27-year-averages are pooled together. These estimates are presented in Table 3 and Figure 2 features the histograms.

Not surprisingly, in view of the previous results, imperfect estimates for MAB produce higher errors in long-run projections than imperfect estimates of SDAB: relative errors of about 0.8 and 0.2 percent, respectively, for the best-guess MAB and SDAB. For a reference, the models' own errors cumulate to about 0.2 percentage points in one generation (excluding the Rectangular and Ryderian models with cumulated errors of about 0.4 percent). Unlike in the short-run, where errors due to imperfect MAB are close to those due to lack of model flexibility, long-run errors caused by approximated MAB are more than twice the own errors of the cruder Rectangular and Ryderian models.

Due to demographic renewal, these errors will cumulate further in the long-run: having projected too high or too low number of births to one generation will change accordingly the size of the parental generation and top up errors for the next generation.

As compared to the effect of assuming imperfect MAB's, the model choice hardly matters in the long run: even the crude Rectangular and Ryderian pentapartite models yield results similar to other models.

Table 3 around here.

Figure 2 around here.

4. Conclusions and implications for projection practices

The above results provide a clear answer to our main research question on relative importance of different ingredients of fertility modeling in population projections. While accuracy of TFR and MAB are highly important for accurate prediction of births, fertility model family (assuming, the model is not too rough) and accuracy of SDAB as well as of higher-order moments have lower impact on projection outcomes.

Lack of knowledge about MAB produces projection errors mounting up to 2.5 cent in a single year or 1.3 percent when births are cumulated over a generation. That is comparable to TFR's mean squared annual change of 4.2 percent in *HFD* data. Country-specific regression of MAB on TFR may improve projection results (bringing the errors down to 1.5 and 0.8 percent on annual and generation bases, respectively). Yet, the correlation between TFR and MAB is so weak, that nearly the same results are obtained by simply setting the MAB to the country-average of the indicator over past data. Given usually high smoothness and inertia of the MAB's temporal change, using the last observed MAB, an average over the recent data, or extrapolating the recent trend may be efficient in short- and medium- run projections.

Uncertainty in SDAB, on the contrary, has rather small effect on projection outcomes, when compared to effects of TFR or MAB uncertainty. When estimated from country-specific regression on TFR, inaccuracy of projections of SDAB contribute only about 0.4 percent errors to annual number of births or 0.2 percent over a generation. These numbers match very well the prediction errors (fractions of a percent, for the most of the years reported) found by Mitra and Romaniuk (1973) for the Pearsonian Type I fertility models with exact values for TFR and MAB. Prediction errors due to uncertain SDAB are of about the same as errors induced by the model structure and is lower in order of magnitude as compared to annual variation of TFR. When the MAB is not set at its empirical value, knowledge on SDAB does not improve the prediction accuracy consistently. Even more, when MAB is estimated from rougher models (HFD-average or HFD-regression over TFR), adding true SDAB to the model worsens the short-term prediction accuracy in most of the cases. This result may indicate that once the MAB is flawed, 'true' SDAB's only add uninformative random variation to the predictions. When the MAB is better known, however (at least, estimated from country-specific averages or regressions), and also in the long-run forecasts, better SDAB's may marginally improve the prediction accuracy. That marginal gain in accuracy, however, would not be sizable as compared to TFR's annual variation.

Similarly, there is relatively small impact of the model choice (and, implicitly, of the higher-order fertility moments neglected in our study) on the projection outcomes. That choice would only make sizable difference when MAB would be known exactly. Even the crudest, Rectangular and Ryderian pentapartite, models produce errors (about 1.8 percent in the short-run, 0.4 percent over a generation) not exceeding those due to imperfect estimates of MAB. All other models' own errors are even smaller and close to each other (about 0.5 percent in the short-run and 0.2 percent over a generation). That said, there are considerable differences in models' accuracy and complexity. Interestingly, our simplest adjustment procedure (Model 2) produces the lowest prediction errors (0.4 percent and 0.19 percent in a year and over a generation, respectively). Although some models show marginally smaller mean squared errors in the long-run, distributions of the errors (Figure 2) suggest this may be due to an uncontrolled selection of the HFD collection of population dynamics. It may be suggested that fertility projections to be based on simple models, such as the transformation (direct or Brass), QS, or Gamma models. In actual projections, one may improve over our roughly assessed standard fertility profiles in Models 2 and 4. This may further lower the prediction errors of the transformation methods. One may also consider a convenient combination of different approaches, when the 'fertility standard' is obtained by one method (e.g., extrapolation of age-specific rates, of the QS or Gamma model's parameters, etc.) and then is transformed by our simple transformation method (as in Models 1, 2) in order to assure exact match to the projected MAB. Such a two-step procedure may, in particular, allow avoiding the resources-consuming optimization procedures in models such as the Brass, QS, or Ryderian models.

All in all, our results have convenient implications for forecasting. One may focus on formulating assumptions for the two key fertility indicators (TFR and MAB) only and pay less attention to other fertility indicators and even to the choice of the model form. Yet, the link between TFR and MAB in processes of fertility postponement and anticipation, but also the remaining need of better theories describing the link (Ní Bhrolcháin 2011) indicate that these parameters may not be projected independently and easily. More research is needed on tempo effects on period fertility and on relations between changes in TFR and MAB.

Indeed, in many projections, especially in the long-run, assumed uncertainty about TFR would exceed its year-to-year change. UN (2011), for example, assumes about ±10 percent range of TFR in the short-run and about ±25 percent range in the long-run (the half-distance between high and low variants as percent of the medium variant). Assuming those ranges cover 95 percent of TFR's expected variation, UN assumptions match to about 5 percent and 12 percent standard errors in projected TFRs in the short- and long-run, respectively. Alders and de Beer (2004) report roughly similar ranges for errors of official TFR forecasts. Compared with these levels of uncertainty in TFR, long-term effects of imperfect predictions of other fertility indicators (even of MAB, but definitely so of SDAB and of the model family) may be neglected in long-term deterministic projections. In the short-run, however, errors due to imperfect MAB may still be large to neglect. Also in the long-run stochastic projections, the relative impact of imperfectness in MAB assumptions may be substantial if effects on population of stochastic annual variations of TFR are averaged and smoothed out.

In deterministic projections, the effects may be further reduced, in the long run, when scenario-based age structures get smoother over time or even converge to a stationary population. Yet, in the short- and medium- run when the effects of original age structure prevail, or in cases when the population asymptotic is not stationary, the projection errors may be as high as reported here. Only in the very long-run and assuming replacement-level fertility, the alternative age patterns of fertility rates may produce similar projected population dynamics. However, there will be substantial cumulated error affecting the total projected population size by that time.

The effect of smoothing age composition will not apply to stochastic projections, where the age composition remains non-stationary along any single random trajectory. Therefore, improving the MAB forecasts or taking the effect of uncertain MAB into account may be an important ingredient of stochastic population projections. A crude way to adjust the stochastic model in the case of uncertain fertility structure may be suggested by errors presented in Table 1 as compared with the TFR's annual change. One may widen the range of uncertainty of TFR (as compared to what is suggested by a time-series analysis) by about six percent when MAB is approximated roughly (e.g., set constant or linked to TFR) (explanation for the adjustment based on the numbers for Model 4: $\frac{\sqrt{4.23^2+1.48^2}}{4.23}\approx 1.06$).

The cohort approach may provide an interesting way out of the problems due to imperfect parameterization of fertility model. Assuming low mortality at childbearing ages, cohort age composition may be assumed flat within childbearing ages. Therefore, in modeling births to cohorts, there would be no need to care about the (usually volatile and uncertain) age structure of cohort fertility as long as the cohort completed fertility is well predicted. Given higher stability of cohort completed fertility compared with the period TFR, projections of the numbers of births to cohorts may be more reliable than usual projections based on period fertility. Unfortunately, this method leaves questions unanswered about timing of births, i.e., about to which calendar years to assign the births to the cohorts. In the short-run, imperfect allocations of births may lead to the same levels of prediction errors as reported here. Those errors may not, however, cumulate over time. In the case when cohort's net reproduction rate is close to one, for example, Ediev (2005, 2007) shows the births' number is inversely proportional to the concurrent MAB; in this case, uncertainty in MAB in one period does not contribute to uncertainty in births in the following periods, as long as the cohorts' reproduction is forecasted correctly. Technically, the cohort approach in population projections may be realized by developing the cohort fertility assumptions and translating them into period fertility rates. Ryder (1989, 1990) proposed a simple method of translation using the tetrapartite fertility model. Our results suggest, however, that this model and even its more complicated versions may be too imprecise in projecting births. Nonetheless, one may use other models, e.g., the Gamma model, to resolve the translation problem along the lines suggested by Ryder.

Our findings may also be of interest in developing the models for fertility reconstruction and decomposition. For the lack of better assumptions, those models used to be based on rather simple age patterns of fertility rates (e.g., Lee 1974, 1985). Our results, also supported by theory (Ediev 2011), suggest these models may be improved by better account for changes in MAB in addition to modeling the change in TFR. Like in the projections, however, other aspects of the fertility model may be simplified in reconstructions and decompositions without substantial consequences for the accuracy of estimates.

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Table 1. Mean squared relative errors of the projected annual number of births for fertility schedules generated at different combinations of the basic parameters, for selected model schedules, percent.

MAB approximation	SDAB approximation	Direct transformation of the fertility schedule	Transformation of the regressions-based fertility schedule vs data	Brass model (empirical standards) vs data	Brass model (regression-based standards) vs data	Schmert-mann's QS model vs data	Gamma model vs data	Rectangular model vs data	Ryderian model vs data
exact value	grand average	0.57	0.79	0.61	0.90	0.88	0.92	1.75	1.95
exact value	country average	0.46	0.65	0.50	0.76	0.80	0.86	1.61	1.92
exact value	regression	0.45	0.63	0.47	0.71	0.72	0.84	1.68	1.94
exact value	country regression	0.32	0.51	0.34	0.60	0.64	0.78	1.59	1.91
grand average	exact value	2.31	2.49	2.43	2.50	2.54	2.59	2.85	3.07
country average	exact value	1.51	1.66	1.62	1.66	1.75	1.80	2.19	2.48
regression	exact value	2.17	2.35	2.30	2.36	2.45	2.49	2.70	2.97
country regression	exact value	1.30	1.47	1.40	1.48	1.55	1.63	2.06	2.36
grand average	grand average	2.19	2.42	2.35	2.45	2.44	2.54	2.84	3.04
country average	country average	1.51	1.69	1.66	1.71	1.79	1.84	2.21	2.51
regression	regression	2.10	2.30	2.26	2.31	2.37	2.46	2.72	2.98
country regression	country regression	1.29	1.47	1.42	1.48	1.54	1.64	2.08	2.38
exact value	**exact value**	**-**	**0.42**	**-**	**0.51**	**.57**	**0.72**	**1.56**	**1.88**
TFR's mean squared annual change (%)		**4.23**							

Table 1 [continuation]. Mean squared relative errors of the projected annual number of births for fertility schedules generated at different combinations of the basic parameters, for selected model schedules, percent.

MAB approximation	**SDAB approximation**	**Schmertmann's QS model vs best-fit QS model**	**Gamma model vs best-fit Gamma model**	**Rectangular model vs best-fit Rectangular model**	**Ryderian model vs best-fit Ryderian model**
exact value	grand average	0.66	0.51	0.74	0.74
exact value	country average	0.55	0.41	0.65	0.64
exact value	regression	0.51	0.40	0.57	0.55
exact value	country regression	0.38	0.29	0.44	0.43
grand average	exact value	2.46	2.25	2.23	2.17
country average	exact value	1.63	1.50	1.48	1.49
regression	exact value	2.33	2.11	2.08	2.04
country regression	exact value	1.42	1.28	1.26	1.27
grand average	grand average	2.35	2.20	2.25	2.20
country average	country average	1.66	1.54	1.66	1.63
regression	regression	2.24	2.09	2.12	2.08
country regression	country regression	1.40	1.30	1.38	1.33
exact value	**exact value**	-	-	-	-

Table 2. Autocorrelations between errors of estimating the number of births for fertility schedules generated at different combinations of the basic parameters (presented numbers are averages over country-specific autocorrelations) **[continued on the next page]**

MAB approximation	SDAB approximation	Direct transformation of the fertility schedule		Transformation of the regressions-based fertility schedule vs data		Brass model (empirical standards) vs data		Brass model (regression-based standards) vs data		Schmertmann's QS model vs data		Gamma model vs data	
		lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years
exact value	grand average	.954	.839	.950	.825	.956	.843	.947	.811	.943	.794	.948	.808
exact value	country average	.947	.820	.945	.806	.948	.820	.941	.790	.936	.775	.944	.793
exact value	regression	.939	.801	.935	.784	.940	.809	.931	.767	.935	.781	.938	.778
exact value	country regression	.906	.732	.922	.755	.900	.724	.919	.739	.923	.759	.934	.763
grand average	exact value	.974	.907	.973	.901	.973	.900	.972	.900	.968	.882	.969	.883
country average	exact value	.966	..880	.967	.883	.966	.878	.967	.883	.962	.860	.962	.858
regression	exact value	.972	.903	.973	.901	.972	.899	.973	.901	.968	.881	.968	.882
country regression	exact value	.952	.848	.962	.867	.953	.847	.962	.867	.959	.852	.956	.844
grand average	grand average	.976	.914	.974	.907	.974	.909	.974	.906	.968	.882	.970	.890
country average	country average	.970	.897	.972	.902	.971	.898	.972	.900	.967	.878	.965	.871
regression	regression	.972	.902	.972	.900	.970	.895	.972	.900	.966	.875	.968	.882
country regression	country regression	.955	.857	.962	.872	.955	.854	.962	.873	.960	.859	.957	.849
Models' own errors		**-**	**-**	**.921**	**.736**	**-**	**-**	**.917**	**.720**	**.931**	**.763**	**.933**	**.749**
TFR's annual change		**.398**	**.258**										

Table 2 [continuation]. Autocorrelations between errors of estimating the number of births for fertility schedules generated at different combinations of the basic parameters (presented numbers are averages over country-specific autocorrelations)

MAB approximation	**SDAB approximation**	**Rectangular model vs data**		**Ryderian model vs data**		**Schmertmann's QS model vs best-fit QS model**		**Gamma model vs best-fit Gamma model**		**Rectangular model vs best-fit Rectangular model**		**Ryderian model vs best-fit Ryderian model**	
		lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years	lag 1 year	lag 2 years
exact value	grand average	.938	.775	.511	.064	.948	.814	.960	.857	.905	.726	.906	.743
exact value	country average	.936	.767	.508	.042	.938	.789	.953	.836	.879	.665	.891	.706
exact value	regression	.930	.748	.511	.047	.933	.790	.944	.822	.902	.719	.893	.718
exact value	country regression	.930	.746	.506	.029	.889	.699	.906	.744	.855	.618	.850	.626
grand average	exact value	.956	.840	.702	.402	.969	.889	.975	.910	.953	.848	.946	.858
country average	exact value	.946	.803	.647	.293	.963	.867	.971	.896	.944	.826	.942	.841
regression	exact value	.956	.842	.702	.412	.968	.887	.974	.908	.951	.845	.947	.858
country regression	exact value	.943	.795	.629	.258	.951	.837	.957	.862	.925	.783	.930	.815
grand average	grand average	.960	.855	.707	.419	.969	.892	.977	.918	.952	.842	.937	.836
country average	country average	.952	.824	.651	.305	.968	.887	.975	.913	.939	.810	.930	.813
regression	regression	.959	.851	.718	.443	.966	.883	.973	.905	.954	.850	.942	.846
country regression	country regression	.946	.806	.638	.275	.953	.848	.959	.870	.929	.791	.926	.807
Models' own errors		**.925**	**.732**	**.494**	**.010**	-	-	-	-	-	-	-	-
TFR's annual change													

Table 3. Mean squared relative errors of projected number of births cumulated over 27 years for fertility schedules generated at different combinations of the basic parameters, for selected models, percent

MAB approximation	SDAB approximation	Direct transformation of the fertility schedule	Transformation of the regressions-based fertility schedule vs data	Brass model (empirical standards) vs data	Brass model (regression-based standards) vs data	Schmert-mann's QS model vs data	Gamma model vs data	Rectangular model vs data	Ryderian model vs data
exact value	grand average	0.26	0.39	0.28	0.45	0.38	0.37	0.59	0.47
exact value	country average	0.23	0.30	0.25	0.34	0.34	0.32	0.48	0.44
exact value	regression	0.20	0.27	0.21	0.30	0.29	0.28	0.46	0.40
exact value	country regression	0.16	0.20	0.18	0.23	0.26	0.25	0.39	0.38
grand average	exact value	1.20	1.26	1.22	1.25	1.28	1.23	1.30	1.26
country average	exact value	0.86	0.93	0.89	0.92	0.95	0.91	0.91	0.91
regression	exact value	1.23	1.30	1.27	1.29	1.33	1.28	1.33	1.30
country regression	exact value	0.78	0.86	0.81	0.85	0.89	0.84	0.84	0.83
grand average	grand average	1.24	1.34	1.29	1.34	1.35	1.31	1.38	1.32
country average	country average	0.94	1.02	0.99	1.02	1.04	1.01	1.03	1.03
regression	regression	1.25	1.32	1.30	1.31	1.36	1.31	1.36	1.33
country regression	country regression	0.80	0.87	0.85	0.87	0.90	0.87	0.87	0.87
exact value	**exact value**	-	**0.19**	-	**0.24**	**0.19**	**0.18**	**0.42**	**0.35**

Table 3 [continuation]. Mean squared relative errors of projected number of births cumulated over 27 years for fertility schedules generated at different combinations of the basic parameters, for selected models, percent

MAB approximation	SDAB approximation	Schmertmann's QS model vs best-fit QS model	Gamma model vs best-fit Gamma model	Rectangular model vs best-fit Rectangular model	Ryderian model vs best-fit Ryderian model
exact value	grand average	0.30	0.25	0.32	0.30
exact value	country average	0.27	0.22	0.27	0.28
exact value	regression	0.22	0.19	0.23	0.21
exact value	country regression	0.19	0.16	0.22	0.19
grand average	exact value	1.24	1.18	1.17	1.17
country average	exact value	0.90	0.85	0.85	0.85
regression	exact value	1.28	1.22	1.22	1.23
country regression	exact value	0.83	0.76	0.75	0.76
grand average	grand average	1.30	1.24	1.24	1.23
country average	country average	0.98	0.94	0.97	0.98
regression	regression	1.30	1.25	1.26	1.26
country regression	country regression	0.84	0.79	0.81	0.82
exact value	**exact value**	-	-	-	-

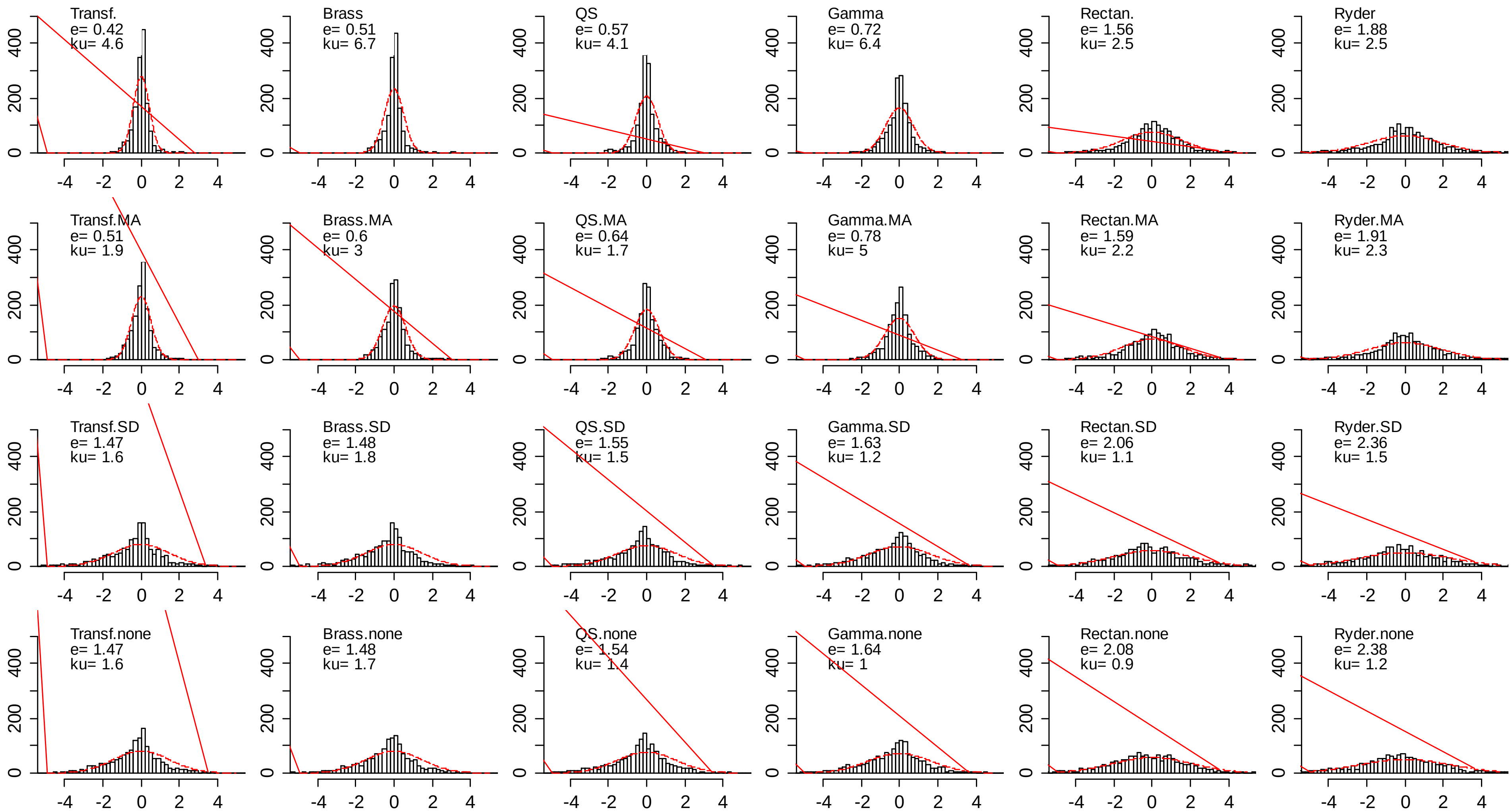

Figure 1. Histograms and corresponding normal distributions for relative prediction errors, selected fertility models (columns 1 to 5: models 2, 4-8), in percent. Rows 1 to 4: MAB and SDAB set at their empirical values; MAB's are empirical and SDAB's are from regression on TFR; SDAB's are empirical and MAB's are from regression on TFR; both parameters are from regression on TFR. All regressions are country-specific. Errors are computed vis-à-vis empirical births. "e" stands for the Mean Squared Relative Error; "ku" stands for the excess kurtosis.

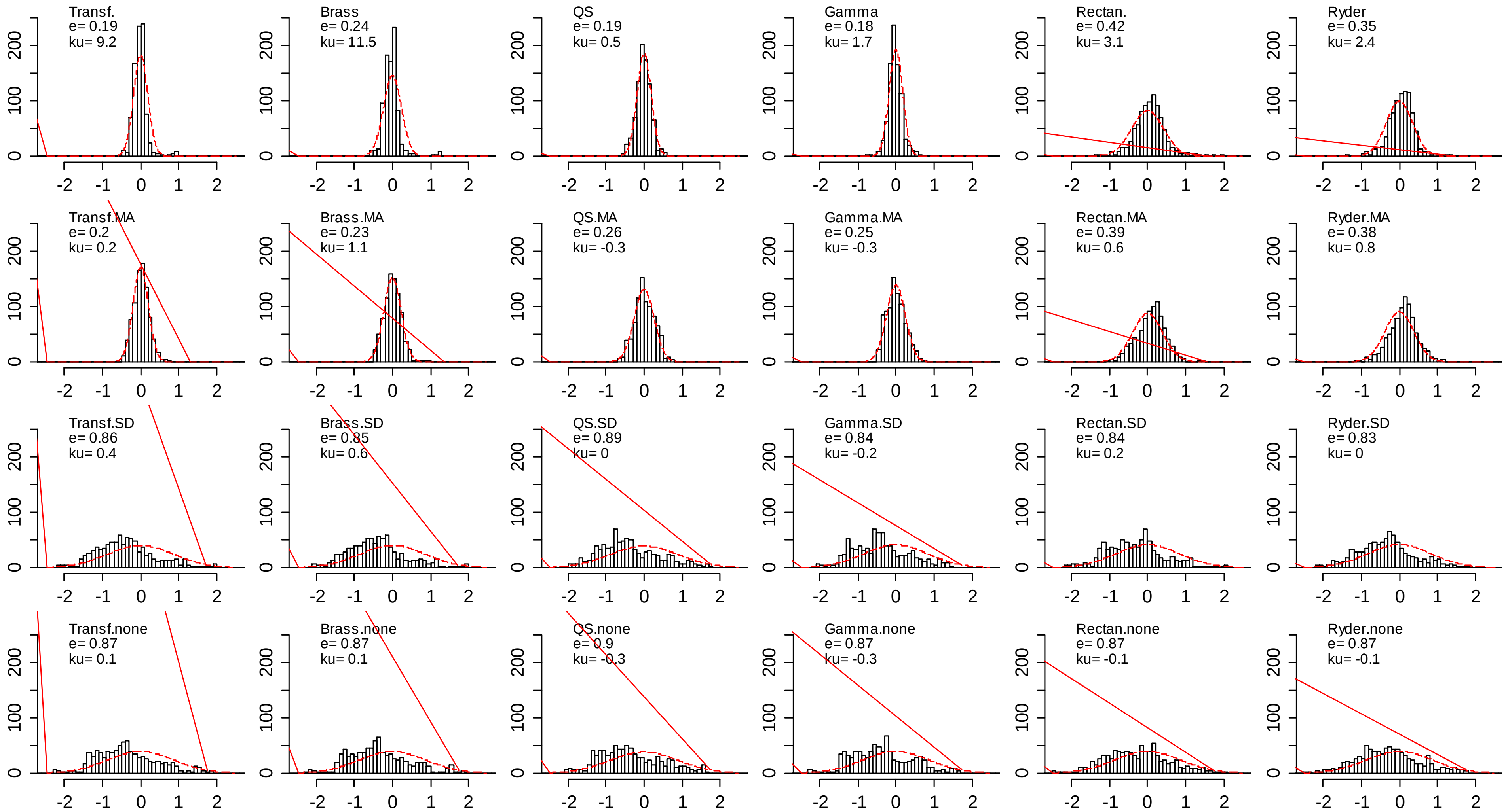

Figure 2. Histograms and corresponding normal distributions for relative prediction errors averaged over periods of 27 years, selected models (columns 1 to 5: models 2, 4-8), in percent. Rows 1 to 4: MAB and SDAB set at their empirical values; MAB's are empirical and SDAB's are from regression on TFR; SDAB's are empirical and MAB's are from regression on TFR; both parameters are from regression on TFR. All regressions are country-specific. Errors are computed vis-à-vis empirical births. "e" stands for the Mean Squared Relative Error; "ku" stands for the excess kurtosis