Equality of Opportunity versus Equality of Outcome in Evaluating Income Redistribution Policies:
Empirical Evidence based on Italian Data

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1. Introduction

This paper presents an empirical analysis of tax-benefit regimes, adopting Equality of Opportunity (EOp) as the evaluation criterion.

The EOp-criterion is a computable concept of equality of opportunity developed by Roemer (1998). This concept is interesting from the policy point-of-view, since the majority of citizens in most industrialised countries, although not unfavourable to redistribution, seem sensible to the way by which a certain outcome has been attained. Redistribution is more likely to receive support if it is designed to correct circumstances that are beyond people's control. On the other hand, if a bad outcome is associated with a lack of effort, redistribution will be much less acceptable.

In a previous contribution that originated from an international research project, this concept has been applied to evaluate the EOp performance of income tax rules in various countries, using a relatively simple common model of labour supply behaviour with calibrated parameters[1]. In this paper we use a previously developed micro-econometric model of household labour supply in Italy[2] together with the EOp-criterion as a basis for evaluating tax regimes.

In Section 2 we give a brief discussion of the justification and definition of the EOp-criterion and its relationship to more traditional concepts of social welfare where the concern focuses upon

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[1] See Roemer et al. (2000).
[2] Aaberge et al. (1999).

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equality of outcome (EO-criterion) rather than equality of opportunity. In the same section we also explain how the EOp-criterion can be generalised to take into account also the equality of outcome.

In Section 3 we use a microeconometric model of household labour supply estimated on 1993 Italian data to simulate the effects of various constant-revenue affine tax rules, i.e. the tax rules defined by a lump-sum transfer (positive or negative) and a constant marginal tax rate that produces the same revenue collected with the observed 1993 rule. These tax rules are evaluated and compared according to the generalised EOp-criterion. Furthermore, the EOp-optimal tax rule is also identified.

In Section 4 we perform a similar exercise as in Section 3, but looking at the class of tax-rules defined by a transfer and two tax rates (instead of one as for the affine rules). In Section 5 we compare the evaluation of tax-rules according the EOp versus the EO criterion. Section 6 summarises the main results. Appendixes A, B and C give essential information on the microeconometric model, on the dataset and on the 1993 tax rule.

2. The EO and EOp *criteria*

The standard approach in evaluating tax systems is to employ a social objective (welfare) function as the basic evaluating instrument. These functions are commonly used to summarise the changes in (adult-equivalent) incomes resulting from introducing various alternatives to the actual tax system in a country. The simplest way to summarise the changes that take place is to add up the income differentials, which means that the individuals are given equal welfare weights independent of whether they are poor or rich. However, if besides the efficiency effects we also care about the distributional consequences of a tax system, then an alternative to the linear additive welfare function is required. In this paper we rely on the rank-dependent social welfare functions that have their origin from Mehran (1976) and Yaari (1987, 1988)[3] and are defined by

$$(2.1) \qquad W_k = \int_0^1 p_k(t) F^{-1}(t) dt, \quad k = 1, 2, \ldots,$$

where F^{-1} is the left inverse of the cumulative distribution function of (adult-equivalent) income with mean μ and $p_k(t)$ is a weight function defined by

$$(2.2) \qquad p_k(t) = \begin{cases} -\log t, & k = 1 \\ \dfrac{k}{k-1}\left(1 - t^{k-1}\right), & k = 2, 3, \ldots. \end{cases}$$

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[3] Several other authors have discussed rationales for this approach, see e.g. Sen (1974), Hey and Lambert (1980), Donaldson and Weymark (1980, 1983), Weymark (1981), Ben Porath and Gilboa (1992) and Aaberge (2000b).

Note that the inequality aversion exhibited by W_k decreases with increasing k. As $k \to \infty$ W_k approaches inequality neutrality and coincides with the linear additive welfare function defined by

$$(2.3) \qquad\qquad W_\infty = \int_0^1 F^{-1}(t)dt = \mu.$$

It follows by straightforward calculations that $W_k \le \mu$ for all j and that W_k is equal to the mean μ for finite k if and only if F is the egalitarian distribution. Thus, W_k can be interpreted as the equally distributed (equivalent) level of equivalent income. As recognised by Yaari (1988) this property suggests that I_k defined by

$$(2.4) \qquad\qquad I_k = 1 - \frac{W_k}{\mu}, \quad k = 1, 2, ...$$

can be used as a summary measure of inequality and moreover proves to be members of the "illfare-ranked single-series Ginis" introduced by Donaldson and Weymark (1980). As noted by Aaberge (2000a) I_1 is actually equivalent to a measure of inequality that was proposed by Bonferroni (1930) whilst I_2 is the Gini coefficient.[4]

Note that each of the welfare functions W_1, W_2, and W_3 and the corresponding measures of inequality (I_1, I_2, and I_3) exhibit aversion to inequality and thus obey the Pigou-Dalton principle of transfers. The essential difference between these welfare functions and the corresponding measures of inequality is revealed by their transfer sensitivity properties. When the transfer sensitivity is judged according to Kolm's principle of diminishing transfers it follows from Aaberge (2000a) that I_1, I_2, and I_3 assign more weight to transfers between persons with a given income difference if these incomes are lower than if they are higher, provided that the income distribution (F) is strictly log-concave, strictly concave and F^2 is strictly concave, respectively. By contrast, when we rely on Mehran's principle of positional transfer sensitivity rather than on the principle of diminishing transfers the results of Aaberge (2000a) show that I_1 satisfies this principle for all distribution functions whereas I_2 and I_3 do not. Note that the principle of positional transfer sensitivity differs from the principle of diminishing transfers by requiring a fixed difference in ranks rather than a fixed difference in incomes. In this case the Gini coefficient (I_2) attaches an equal weight to a given transfer irrespective of whether it takes place at the lower, the middle or the upper part of the income distribution, whilst I_3 assigns more weight to transfers at the upper than at the middle and the lower part of the income distribution. Roughly spoken, this means that I_1 exhibits very high downside inequality aversion and is particular

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[4] For further discussion of the family $\{I_k : k=1, 2, ...\}$ of inequality measures we refer to Mehran (1976), Donaldson and Weymark (1980, 1983), Bossert (1990) and Aaberge (2000a).

sensitive to changes that concern the poor part of the population, whilst I_2 normally pays more attention to changes that take place in the middle part of the income distribution. The I_3-coefficient exhibits upside inequality aversion and is thus particularly sensitive to changes that occur in the upper part of the income distribution.

For a given sum of income the welfare functions W_1, W_2, and W_3 take their maximum value when everyone gets the same income and may thus be interpreted as EO-criteria (equality of outcome) when employed as a measure for judging between tax systems.

However, as indicated by Roemer (1998) the EO-criterion is controversial and suffers from the drawback of receiving little support among citizens in a nation.[5] This is simply due to the fact that differences in outcomes resulting from differences in efforts are in general considered ethically acceptable and thus should not be the target of a redistribution policy. An egalitarian redistribution policy should instead seek to equalise those income differentials for which the individuals should not be held responsible, because they were beyond their control. Thus, not only the outcome, but its origin and how it was obtained, matters. This is the essential idea behind Roemer's (1998) theory of equality of opportunity where people are supposed to differ with respect to *circumstances*.

Circumstances are attributes of the environment of the individual that influence the earnings potential of the individual, and which are "beyond his control".

This study defines circumstances by family background that classify the individuals into three types according to father's years of education:

- less than 5 years (Type 1),
- 5-8 years (Type 2)
- and more than 8 years (Type 3).

Assume that $F_j^{-1}(t)$ is the income level of the individual located at the tth quantile of the income distribution (F_j) of type j. The differences in incomes within each type are assumed to be due to different degrees of effort for which the individual is to be held responsible, whereas income differences that may be traced back to family background are considered to be "beyond the control" of the individual. As indicated by Roemer (1998) this suggests that we may measure a person's effort by the quantile of the income distribution where he is located. Next, Roemer declares that two individuals in different types have expended the same degree of effort if they have identical positions (rank) in the income distribution of their type. Thus, an EOp (Equality of Opportunity) tax policy should aim at designing a tax system such that $\min F_j^{-1}(t)$ is maximised for each quantile t. However, since this criterion is rather demanding and in most cases probably will not produce a complete ordering of the

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[5] See also Dworkin (1981a, 1981b), Arneson (1989, 1990), Cohen (1989) and Roemer (1993).

tax systems under consideration a weaker ranking criterion is required. To this end Roemer (1998) proposes to employ as the social objective the average of the lowest income at each quantile,

$$(2.5) \qquad \tilde{W}_\infty = \int_0^1 \min_j F_j^{-1}(t)dt$$

Thus, $\tilde{W}_\infty$ ignores income differences *within* types and is solely concerned about differences that arise from differential circumstances. By contrast, the EO criteria defined by (2.1) do not discern between the different sources that contribute to income inequality and thus account for inequality within as well as between types. As an alternative to (2.1) and (2.5) we introduce the following extended family of EOp welfare functions,

$$(2.6) \qquad \tilde{W}_k = \int_0^1 p_k(t)\min_j F_j^{-1}(t)dt, \ \ k=1,2,...,$$

where $p_k(t)$ is defined by (2.2).

The essential difference between $\tilde{W}_k$ and $\tilde{W}_\infty$ is that $\tilde{W}_k$ gives increasing weight with decreasing quantile to the income differentials between types. Thus, in this respect $\tilde{W}_k$ captures also an aspect of inequality *within* types. As explained above, the concern for *within* inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of $\min_j F_j^{-1}(t)$.

Note that $\min_i F_i^{-1}(t)$ defines the inverse of the following cumulative distribution function $\left(\tilde{F}\right)$

$$(2.7) \qquad \tilde{F}(x) = \Pr\left(\tilde{F}^{-1}(T) \le x\right) = \Pr\left(\min_i F_i^{-1}(T) \le x\right) = 1 - \prod_i \left(1 - F_i(x)\right),$$

where T is a random variable with uniform distribution function (defined on [0,1]). Thus, we may decompose the EOp welfare functions $\tilde{W}_k$ in a similar way as was demonstrated for the EOp welfare functions W_k. Accordingly, we have that

$$(2.8) \qquad \tilde{W}_k = \tilde{W}_\infty \left(1 - \tilde{I}_k\right), \ \ k=1,2,...$$

where $\tilde{I}_k$ defined by

$$(2.9) \qquad \tilde{I}_k = 1 - \frac{\tilde{W}_k}{\tilde{W}_\infty}, \ \ k=1,2,...$$

is a summary measure of inequality for the mixture distribution $\tilde{F}$.

Expression (2.8) demonstrates that the EOp welfare functions $\tilde{W}_k$ for $k < \infty$ take into account value judgements about the trade-off between the mean income and the inequality in the distribution of income for the most EOp disadvantaged people. Thus, $\tilde{W}_k$ may be considered as an inequality within type adjusted version of the pure EOp welfare function that was introduced by Roemer (1998). As explained above, the concern for within inequality is greatest for the most disadvantaged type, i.e. for the type that forms the largest segment(s) of the mixture distribution $\tilde{F}$. Alternatively, $\tilde{W}_k$ for $k < \infty$ may be interpreted as an EOp welfare function that, in contrast to $\tilde{W}_\infty$, gives increasing weight with decreasing quantile to the income differentials between types.

3. Microeconometric simulation of tax reforms

The purpose of this section is to make an EOp-evaluation of the 1993 Italian tax system and various alternative two-parameter and three-parameter tax rules under the constraint of fixed tax revenue. To this end we employ the labor supply model(s) and simulation framework explained in Appendix A to simulate the labor supply behavior of single females, single males and couples that are between 18 and 54 years old. To capture the heterogeneity in preferences we have estimated three separate models of labor supply; one for single females, one for single males and one for couples. Note, however, that the condition of constant tax revenue concerns the total population of individuals at the ages between 18 and 54. Thus, the tax paid by each of the groups is treated as an endogenous variable in the simulation exercises. The main features of the 1993 tax rule are briefly illustrated in Appendix C.

The tax reform simulations consists of five main steps:

1. The tax rule is applied to individual earners' gross incomes in order to obtain disposable incomes. New labour supply responses in view of a new tax rule are taken into account by the household labour supply models for singles and couples briefly described in Appendix A. The computation of taxes is made under the conditions of constant tax revenue and non-negative disposable household incomes.

2. To each decision maker individuals between 18 and 54 years old, an *equivalent income* is imputed, computed as total disposable household income divided by the square root of the number of household components.

3. We then build the individual equivalent income distributions F_1, F_2 and F_3 for the types defined according to parental (actually father's) education: less than 5 years (type 1), 5-8 years (type 2) and more than 8 years (type 3).

4. Last, we compute $\tilde{W}_k$ for $k = 1, 2, 3$ and ∞ defined by (2.6).

5. The above steps are iterated in order to find the tax rule (c, t) that produces the highest value of $\tilde{W}_k$ for each value of k under the constraint of unchanged tax revenue.

3.1. EOp-evaluation of alternative two-parameter tax rules

The alternative two-parameter tax rules are of the following type: $x = c + (1-t)y$, where

y = gross income,

x = disposable income,

c = lump-sum transfer (positive or negative),

t = constant marginal tax rate.

Note that the income and tax figures below are measured in 1000 ITL. The results of the two-parameter tax reform simulations are summarised in Tables 1 and 2 and in Figure 1.

Table 1. EOp-optimal two-parameter tax systems under various social objective criteria $\left(\tilde{W}_k\right)$

k	1	2	3	∞
marginal tax rate, t	.774	.637	0	0
lump-sum tax/transfer, c	11,500	9,500	-5,790	-5,790

Table 1 presents the EOp-optimal affine tax rules for different values of k, i.e. for different degrees of concern for *within* inequality. Remember that the higher is k, the lower is the concern for within inequality. As k becomes very large, it approaches a pure EOp criterion, i.e. the case in which the social planner cares about between inequality but not about within inequality; this case is associated with the last column. On the other hand, $k = 1$ (first column) is the case with most concern about within inequality.

As demonstrated by Table 1 the results are very dependent on the value of k. For $k \geq 3$, the EOp-optimal tax rule is the pure lump-sum tax (i.e. $t = 0$ and $c < 0$) whereas for $k \leq 2$ the optimal tax rule consists of a very high marginal tax rate and a positive lump-sum transfer. An implication is that the concern for the equality of opportunity by itself does not imply high marginal tax rates. Only if we also account for within inequality, the optimal policy may entail high marginal tax rates.

Figure 1. Distributions of observed equivalent income by type. 1000 ITL
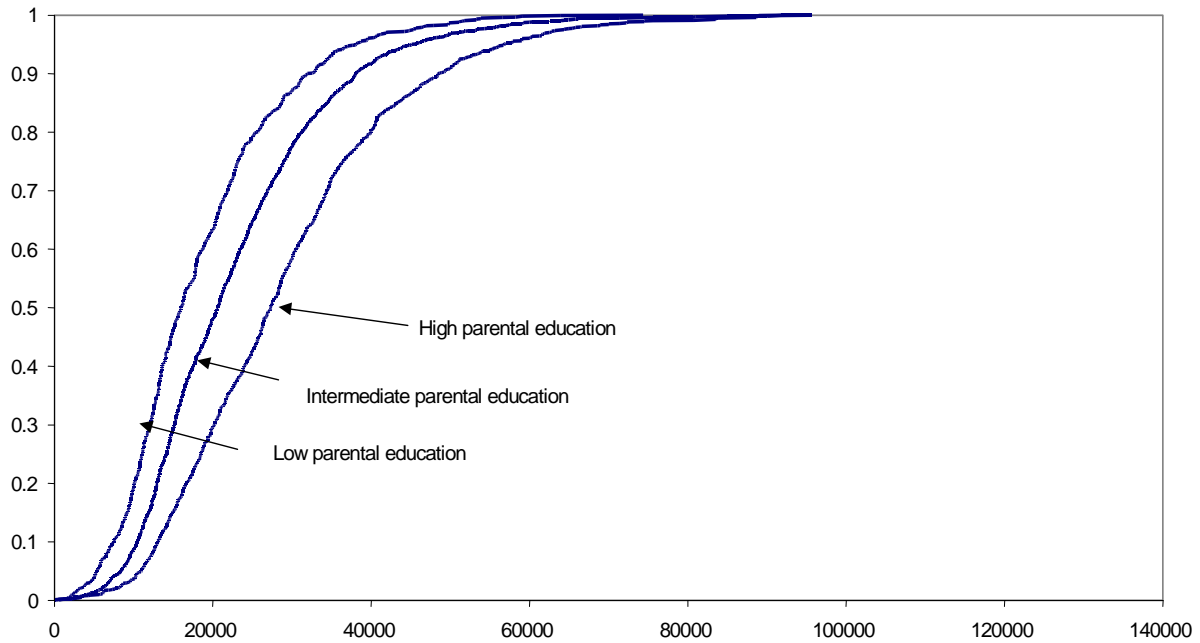
Figure 2. Distributions of individual equivalent income by type under the EOp1(1) and EOp1(3) tax systems. 1000 ITL
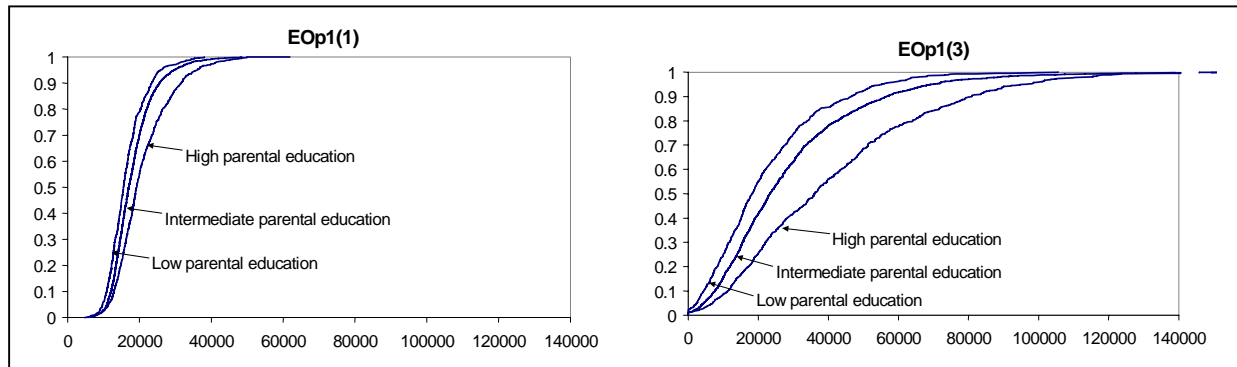

Table 2 and Figures 1 and 2 give more details. The grahps illustrate the equivalent income distributions under the actual 1993 tax rule (Figure 1) and under the EOp-optimal rules for k=1 and k≥3 (Figure 2). Table 2 reports the value of the EOp criterion for different tax rules. In particular, we focus on the comparison between the observes rule (1993), the pure flat tax (a theoretical benchmark, and the three linear rules that are EOp optimal under different values of k. In each column (i.e. for each k) the bold figure is the maximised value of the EOp criterion, i.e. it corresponds to the EOp-optimal tax rule. EOp1(r) denotes the EOp-optimal affine tax rule when k = r.

Table 2. EOp-performance $\left(\tilde{W}_k\right)$ of the 1993 tax system, a flat tax system and three different EOp-optimal two-parameter tax systems

Tax system	Social objective function $\left(\tilde{W}_k\right)$	k			
		1	2	3	∞
1993 tax system		10,523	12,797	13,893	18,323
Flat tax	$\begin{pmatrix} t=.181 \\ c=0 \end{pmatrix}$	10,834	13,496	14,823	20,449
EOp1 (1)	$\begin{pmatrix} t=.774 \\ c=11,500 \end{pmatrix}$	**12,661**	13,652	14,077	15,641
EOp1 (2)	$\begin{pmatrix} t=.637 \\ c=9,500 \end{pmatrix}$	12,406	**13,660**	14,237	16,486
EOp1 (3)	$\begin{pmatrix} t=0 \\ c=-5,790 \end{pmatrix}$	9,942	13,270	**14,992**	**22,231**

Table 2 allows to compare the EOp performance of the various rules for a given k (note that the comparison only makes sense between elements of the same column). We can see that although the flat tax is never EOp-optimal, yet for any value of k it improves upon the observed 1993 rule. More generally, one can always find an affine tax rule that is EOp-preferred to the observed 1993 one. However, the direction along which one can find EOp-optimal tax rules depends crucially on the value of k. If k = 1 one has to move towards very high marginal tax rates (coupled with high transfers). If k is greater than 1, then the EOp-optimal tax rules require lower marginal tax rate – and more revenue collected through the lump-sum part of the tax. These aspects are further illustrated by Figure 1, where we draw the locus – in the (t,c) plane – of the revenue-constant affine tax rules, and for any k we indicate the sets of tax rules with a lower or with a higher EOp performance with respect to the observed rule. As k increases the graphs in Figure 1 demonstrate that the more we reduce the marginal tax rate – and the more revenue we collect through lump-sum taxation – the better is the EOp-performance.
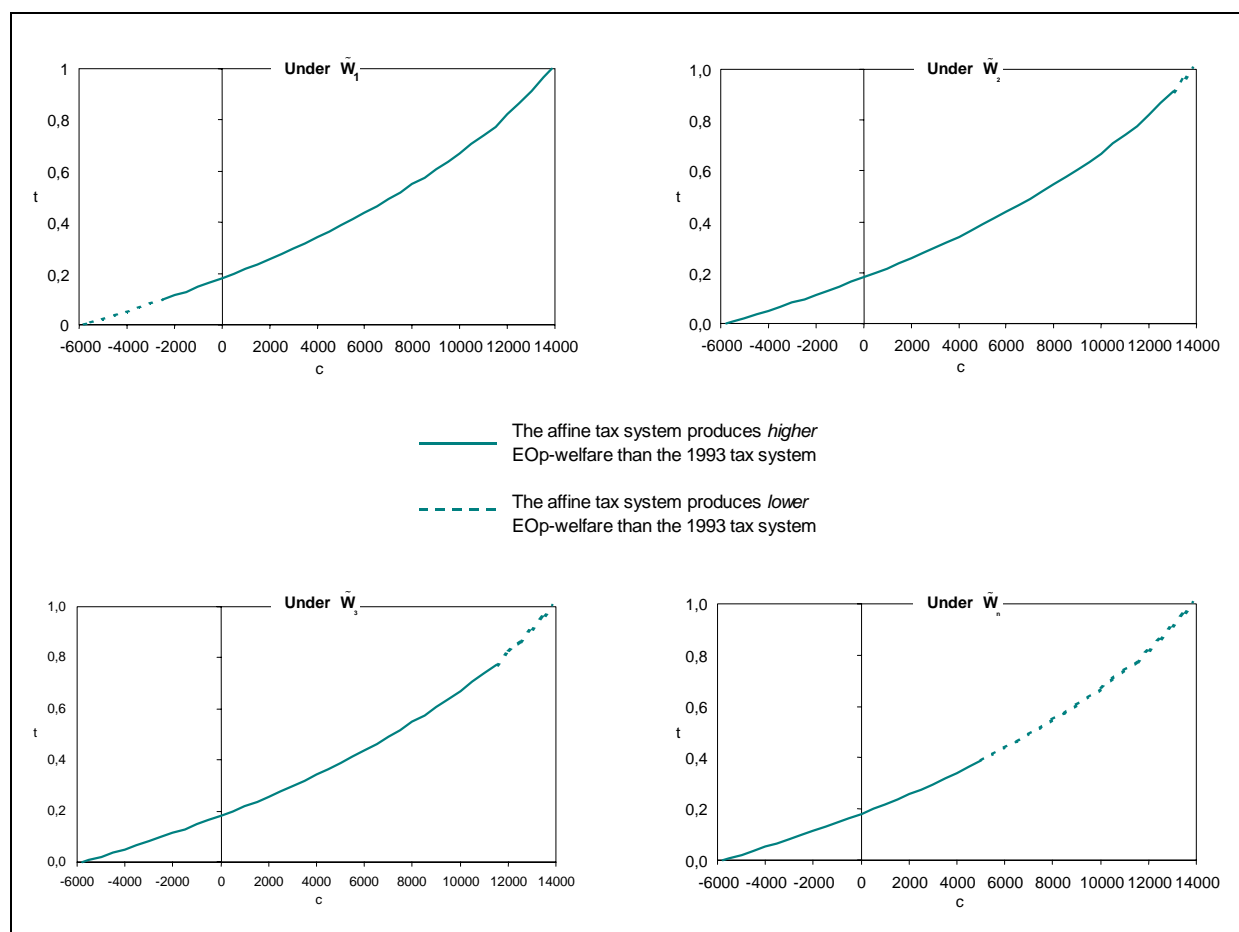
The fact that the optimal tax rule is the pure lump-sum tax provided we do not put too much weight on *within* inequality is a somewhat striking result in itself. After all, EOp is an egalitarian criterion, and one would expect it to favour heavier marginal taxation. How can we justify this apparently counter-intuitive result? A possible explanation resides into the relatively high labour supply response among the least advantaged individuals. Since the Eop-criterion requires to maximise a weighed average of the incomes of the least advantaged type, and since the labour supply of these individuals turn out to be very responsive to higher net wage rates, it follows that lower marginal tax

rates (or, in the limit, a marginal tax rate equal to 0) can in fact improve substantially the welfare of this group, unless the weighting is such that the inequality *within* the group can counterbalance the higher efficiency attained by the group on average. Table 4 gives some support to the above argument by illustrating the labour supply response of the different types when facing alternative tax rules. When the pure lump-sum tax is applied, the labour supply (and therefore the available income) of group 1 (the most disadvantaged group) increases much more (as percentage variation) than labour supply of groups 2 or 3.

Table 3. Decomposition of EOp social welfare $\left(\tilde{W}_k \right)$ under the 1993 tax system, a flat tax system and various EOp-optimal two-parameter tax systems

Tax system	$\tilde{W}_\infty$	Measure of inequality		
		$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$
1993 tax system	18,323	.426	.302	.242
Flat tax $\begin{pmatrix} t = .181 \\ c = 0 \end{pmatrix}$	20,449	.470	.340	.275
EOp1 (1) $\begin{pmatrix} t = .774 \\ c = 11,500 \end{pmatrix}$	15,642	.191	.127	.100
EOp1 (2) $\begin{pmatrix} t = .637 \\ c = 9,500 \end{pmatrix}$	16,486	.247	.171	.136
EOp1 (3) $\begin{pmatrix} t = 0 \\ c = -5,790 \end{pmatrix}$	22,231	.553	.403	.326

Figure 3. Sets of revenue constant affine tax systems under different EOp welfare criteria $\left(\tilde{W}_k\right)$

The affine tax system produces *higher* EOp-welfare than the 1993 tax system

The affine tax system produces *lower* EOp-welfare than the 1993 tax system

Table 4. Labour supply by types under different tax systems*

Tax system	All	Type		
		1	2	3
1993 tax system	1383	1279	1383	1469
Flat tax $\begin{pmatrix} t = .181 \\ c = 0 \end{pmatrix}$	1391 *(+0.58)*	1369 *(+7.04)*	1362 *(+1.52)*	1471 *(+0.14)*
EOp1 (1) $\begin{pmatrix} t = .774 \\ c = 11,500 \end{pmatrix}$	1095 *(-20.82)*	1109 *(-13.29)*	1087 *(-21.40)*	*1100 (-25.12)*
EOp1 (2) $\begin{pmatrix} t = .637 \\ c = 9,500 \end{pmatrix}$	1160 *(-16.12)*	1142 *(-10.71)*	1148 *(-16.99)*	*1200 (-18.31)*
EOp1 (3) $\begin{pmatrix} t = 0 \\ c = -5,790 \end{pmatrix}$	1487 *(+7.52)*	1450 *(+13.37)*	1459 *(+5.50)*	1578 *(+7.42)*

*Percentage changes relative to the labour supply under the 1993 tax system in parentheses.

Table 5. Relative propositions, mean observed individual (disposable equivalent) income (W_{obs}) and changes in mean individual income $(W_{\infty} - W_{obs})$ when the tax regime is changed to lump-sum taxation by gender, family status, economic status (poverty) and family background (type). In 1000 Lire

Individual and household characteristics	Household type (by family background)	1	2	3	All
All	Proportion (per cent)	20.3	54.7	25.0	100
	Mean income	21,107	22,831	29,312	23,540
	Changes in mean income	3,907	5,794	12,011	6,969
Single males	Proportion (per cent)	19.9	51.7	28.4	100
	Mean income	22,369	28,480	34,046	28,843
	Changes in mean income	3,210	7,013	7,343	6,350
Single females	Proportion (per cent)	15.8	51.7	32.6	100
	Mean income	18,076	20,110	26,085	21,734
	Changes in mean income	3,134	2,568	4,412	3,258
Two person households	Proportion (per cent)	15.3	51.2	33.5	100
	Mean income	24,377	28,613	33,913	29,741
	Changes in mean income	7,153	9,781	14,909	11,097
Three person households	Proportion (per cent)	16.5	55.0	28.5	100
	Mean income	20,091	24,795	29,050	25,235
	Changes in mean income	4,678	5,066	14,333	7,648
Households with more than three persons	Proportion (per cent)	23.5	55.8	20.7	100
	Mean income	16,848	20,516	27,349	21,064
	Changes in mean income	3,022	5,153	9,785	5,608
Poor individuals	Proportion (per cent)	39.2	50.4	10.4	100
	Mean income	7,235	7,720	7,424	7,500
	Changes in mean income	5,276	7,487	13,174	7,216
Non-poor individuals	Proportion (per cent)	18.0	55.2	26.8	100
	Mean income	21,320	24,541	30,368	25,528
	Changes in mean income	3,537	5,603	11,955	6,939

3.2. EOp-evaluation of alternative three-parameter tax rules

One might suspect that the results – in particular the EOp-optimality of a pure lump-sum tax for k = 3 or greater – are somewhat forced by the fact that we restrict the simulation to the class of two-parameter tax rules. Since the disadvantaged individuals are more responsive – in terms of labour supply – than the rich and/or advantaged individuals, we should be able to improve upon the pure lump-sum tax or upon the high marginal rate rules, by adopting a three-parameter tax rule. Here we explore this policy direction. The class of tax rules considered is defined as follows:

$$x = \begin{cases} c + (1-t_1)y & \text{if } y \leq \overline{y} \\ c + (1-t_1)\overline{y} + (1-t_2)(y-\overline{y}) & \text{if } y > \overline{y} \end{cases}$$

where

x = disposable income,

y = gross income,

$\overline{y}$ = average individual gross income in Italy on the survey year (1993).

Table 6 reports the optimal three-parameter rules for different values of k. For example, for k=1 the optimal rule is defined by a transfer c = 12,500, a first marginal tax rate t_1 = 0.856 and a second marginal tax rate t_2 = 0.776. By comparing Table 6 with Table 1, we see that for k = 1 and k = 2 the EOp-optimal rules differ quite markedly depending on whether one considers a two-parameter (Table 1) or a three-parameter rule (Table 6). However, when we employ the pure EOp-welfare function (k = ∞) we are back to the EOp-optimality of the pure lump-sum tax.

The comparison of Table 7 with Table 2 adds more details to the picture. EOp2(r) denotes the EOp-optimal three-parameter tax rule when k = r.

Table 6. EOp-optimal three-parameter tax systems under various social objective criteria $\left(\tilde{W}_k \right)$

k	1	2	3	∞
t_1	.856	.251	0	0
t_2	.776	.531	.168	0
c	12,500	3,500	-3,500	-5,790

Table 7. EOp-performance $\left(\tilde{W}_k\right)$ of the 1993 tax system, a flat tax system and three different EOp-optimal three-parameter tax systems

Tax system	Social objective function $\left(\tilde{W}_k\right)$	K			
		1	2	3	∞
1993 tax system		10,523	12,797	13,893	18,323
EOp2 (1)	$\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{pmatrix}$	**12,685**	13,606	13,988	15,393
EOp2 (2)	$\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{pmatrix}$	11,769	**13,822**	14,789	18,508
EOp2 (3)	$\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{pmatrix}$	10,636	13,649	**15,135**	21,156
EOp2 (∞)	$\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5,790 \end{pmatrix}$	9,942	13,270	14,992	**22,231**

Table 8. Decomposition of EOp social welfare $\left(\tilde{W}_k\right)$ under various three-parameter tax systems

Tax system	$\tilde{W}_\infty$	Measure of inequality		
		$\tilde{I}_1$	$\tilde{I}_2$	$\tilde{I}_3$
1993 tax system	18,323	.426	.302	.242
EOp2 (1) $\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{pmatrix}$	15,393	.176	.116	.091
EOp2 (2) $\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{pmatrix}$	18,508	.364	.253	.201
EOp2 (3) $\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{pmatrix}$	21,156	.497	.355	.285
EOp2 (∞) $\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5,790 \end{pmatrix}$	22,231	.553	.403	.326

4. Comparison of empirical results based on EOp and EO criteria

In this section we focus upon the evaluation of the EOp-optimal policies (illustrated in Section 3 and 4) using the more traditional evaluation criterion of equality of outcome (EO criterion, see Section 2). Table 9 reports the EO-performance i.e. the level of the EO social welfare function (defined in Section 2) of five policies discussed above for various values of k. The policies are the observed 1993 tax rule, the flat tax, and the three EOp-optimal two-parameter rules for $k = 1$, $k = 2$ and $k \geq 3$. Table 10 shows the decomposition of the EO-criterion into the efficiency and the inequality terms. The exercise is repeated, with similar results, for the EOp-optimal three-parameter tax rules (Tables 11 and 12). More generally, we have also searched for the EO-optimal rule within the whole classes of the two-parameter and three-parameter tax rules, and it always turns out that the pure lump-sum tax is optimal whatever the value of k. Thus, if we do not explicitly account for inequality between types in the EOp-manner, the optimal policy always implies a zero marginal tax rate (coupled with a positive lump-sum tax), *whatever the degree of inequality aversion*. Table 12 clarifies that this result is due to very large efficiency effects of the lump-tax rule, large enough to over-compensate the also large inequality effects.

Table 9. EO-performance (W_k) of the 1993 tax system, a flat tax system and three different EOp-optimal two-parameter tax systems

Tax system	Social objective function (W_k)	K			
		1	2	3	∞
1993 tax system		13,747	16,586	17,967	23,540
Flat tax	$\begin{pmatrix} t = .181 \\ c = 0 \end{pmatrix}$	13,790	17,180	18,893	26,581
EOp1 (1)	$\begin{pmatrix} t = .774 \\ c = 11,500 \end{pmatrix}$	13,445	14,546	15,028	16,867
EOp1 (2)	$\begin{pmatrix} t = .637 \\ c = 9,500 \end{pmatrix}$	13,653	15,100	15,775	18,534
EOp1 (3)	$\begin{pmatrix} t = 0 \\ c = -5,790 \end{pmatrix}$	13,901	18,260	20,522	30,510

Table 10. Decomposition of social welfare $\left(W_k \right)$ with respect to mean and income inequality under the 1993 tax system, a flat tax system and various two-parameter tax systems

Tax system	Mean income	Measure of inequality		
		I_1	I_2	I_3
1993 tax system	23,540	.416	.295	.237
Flat tax $\begin{pmatrix} t=.181 \\ c=0 \end{pmatrix}$	26,581	.481	.354	.289
EOp1 (1) $\begin{pmatrix} t=.774 \\ c=11,500 \end{pmatrix}$	16,867	.203	.138	.109
EOp1 (2) $\begin{pmatrix} t=.637 \\ c=9,500 \end{pmatrix}$	18,534	.263	.185	.149
EOp1 (3) $\begin{pmatrix} t=0 \\ c=-5,790 \end{pmatrix}$	30,510	.544	.402	.327

Table 11. EO-performance $\left(W_k \right)$ of three different EOp-optimal three-parameter tax systems

Tax system \ Social objective function $\left(W_k \right)$		K			
		1	2	3	∞
1993 tax system		13,747	16,586	17,967	23,540
EOp2 (1)	$\begin{pmatrix} t_1=.856 \\ t_2=.776 \\ c=12,500 \end{pmatrix}$	13,361	14,400	14,846	16,560
EOp2 (2)	$\begin{pmatrix} t_1=.251 \\ t_2=.531 \\ c=3,500 \end{pmatrix}$	13,660	16,010	17,119	21,477
EOp2 (3)	$\begin{pmatrix} t_1=0 \\ t_2=.168 \\ c=-3,500 \end{pmatrix}$	13,825	17,577	19,455	27,573
EOp2 (∞)	$\begin{pmatrix} t_1=t_2=0 \\ c=-5,790 \end{pmatrix}$	13,901	18,260	20,522	30,510

Table 12. Decomposition of social welfare (W_k) with respect to mean and income inequality under different tax systems

Tax system	Mean income	Measure of inequality		
		I_1	I_2	I_3
1993 tax system	23,540	.416	.295	.237
EOp2 (1) $\begin{pmatrix} t_1 = .856 \\ t_2 = .776 \\ c = 12,500 \end{pmatrix}$	16,560	.193	.130	.104
EOp2 (2) $\begin{pmatrix} t_1 = .251 \\ t_2 = .531 \\ c = 3,500 \end{pmatrix}$	21,477	.364	.255	.203
EOp2 (3) $\begin{pmatrix} t_1 = 0 \\ t_2 = .168 \\ c = -3,500 \end{pmatrix}$	27,573	.499	.363	.294
EOp2 (∞) $\begin{pmatrix} t_1 = t_2 = 0 \\ c = -5,790 \end{pmatrix}$	30,510	.544	.402	.327

6. Conclusions

We have used a microeconometric model of household labour supply in Italy in order to simulate alternative tax rules within the classes of two- and three-parameter tax rules. We have adopted the criterion of Equality of Opportunity as developed by Roemer (1998) for evaluating the effects on income distribution. We have also offered a generalisation of the EOp criterion that permits to complement the pure EOp criterion with a variable degree of aversion to inequality *within* types. A rather striking result is that the optimal tax rule turns out to be the pure lump-sum tax, under the pure EOp criterion or with moderate (k greater than 2) degrees of aversion to inequality *within* types. A high degree (k less than 2) of inequality aversion instead commands EOp-optimal rules with non-zero marginal tax rates. On the other hand, when using the EO criterion, the pure lump-sum always turns out to be the optimal one, at least with respect to the classes of two- and three-parameter rules. The result seems to depend on a relatively high labour supply response from the most disadvantaged group: the labour supply incentives – and the efficiency effects for the most disadvantaged – generated by the pure lump-sum tax are large enough to overcome the disequalising effects obtained within the group. Further work is required to explore the performance of tax rules that are more complex than the two- or three-parameter rules.

Appendix A

The microeconometric model and the simulation procedure

The model used draws upon the framework introduced by Dagsvik (1994). The agents choose among jobs, each job being defined by a wage rate *w*, hours of work *h* and other characteristics *j*. For expository simplicity we consider in what follows a single person household, although the model we estimate considers both singles and couples. The problem solved by the agent looks like the following:

(A.1)
$$\max_{(x,h,j)\in B} U\left(x,h,j\right)$$

under the budget constraint $x = f\left(wh, m\right),$ where

h = hours of work

w = gross wage rate

j = other job and/or household characteristics

m = gross exogenous income

x = disposable income

$f(.,.)$ = tax rule that transforms gross incomes (wh,m) into net income *x*.

The set B is the opportunity set, i.e. it contains all the opportunities available to the household. For generality we also include non-market opportunities into B; a non-market opportunity is a "job" with $w = 0$ and $h = 0$. Agents can differ not only in their preferences and in their wage (as in the traditional model) but also in the number of available jobs of different type. Note that for the same agent, wage rates (unlike in the traditional model) can differ from job to job. As analysts we do not know exactly what opportunities are contained in *B*. Therefore we use a probability density function to represent *B*. Let us denote with $p(h,w)$ the density of jobs of type $(h,w).$ By specifying a probability density function on *B* we can for example allow for the fact that jobs with hours of work in a certain range are more or less likely to be found, possibly depending on agent's characteristics; or for the fact that for different agents the relative number of market opportunity may differ. From expression (A.1) it is clear that what we adopt is a choice model; choice, however, is constrained by the number and the characteristics of jobs in the opportunity set. Therefore the model is also compatible with the case of involuntary unemployment, i.e. an opportunity set that does not contain any market opportunity; besides this extreme case, the number and the characteristics of market (and non-market) opportunities

in general vary from individual to individual. Even if the set of market opportunities is not empty, in some cases it might contain very few elements and/or elements with bad characteristics.

We assume that the utility function can be factorized as

(A.2) $$U\big(f(wh,m),h,j\big) = V\big(f(wh,m),h\big)\varepsilon(h,w,j)$$

where V and ε are the systematic and the stochastic component respectively, and ε is i.i.d. according to:

(A.3) $$\Pr\big(\varepsilon \leq u\big) = \exp\big(-u^{-1}\big)$$

The term ε is a random taste-shifter which accounts for the effect on utility of all the characteristics of the household-job match which are observed by the household but not by us. We observe the chosen h and w. Therefore we can specify the probability that the agent chooses a job with observed characteristics (h,w). It can be shown that under the assumptions (A.1), (A.2) and (A.3) we can write the probability density function of a choice (h,w) as follows[6]:

(A.4) $$\varphi(h,w) = \frac{V\big(f(wh,m),h\big)\,p(h,w)}{\iint\limits_{q,z} V\big(f(zq,m),q\big)\,p(q,z)\,dq\,dz}$$

Expression (A.4) is analogous to the continuous multinomial logit developed in the transportation and location analysis literature. The intuition behind expression (A.4) is that the probability of a choice (h,w) can be expressed as the relative attractiveness – weighted by a measure of "availability" $p(h,w)$ – of jobs of type (h, w). More details on the derivation of (A.4) can be found in Aaberge et al. (1999).

From (A.4) we also see that this approach does not suffer from the complexity of the tax rule f. The tax rule, however complex, enters the expression as it is, and there is no need to simplify it in order to make it differentiable or manageable as in the traditional approach. The crucial difference is that in the traditional approach the functions representing household behavior are derived on the basis of a comparison of marginal variations of utility, while in the approach that we follow a comparison of levels of utility is directly involved.

In order to estimate the model we choose convenient but still flexible parametric forms for V and $p(h,w)$. The parameters are estimated by maximum likelihood. The likelihood function is the

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[6] See Aaberge et al. (1999).

product of the choice densities (A.4) for every household in the sample. We refer to Aaberge et al. (1998, 2001) for the estimated parameters[7].

Once the parameters have been estimated, we can simulate the effects of different tax rules. Then we can evaluate the effect of a new rule f* by solving the new problem:

(A.5)
$$\max_{(h,w,j)\in B} V\left(f^*\left(wh,m\right),h,j\right)\varepsilon\left(h,w,j\right).$$

As a practical matter, the simulation procedure works as follows. First, for each household we simulate the opportunity set with 200 points: one is the chosen alternative, the other 199 are built by drawing from the estimated p(h,w) density. Second, for each household and each point in the opportunity set we draw a value ε from the distribution (A.3). Third, for each household we solve problem (A.5).

For further details on the empirical specification and the estimation results we refer to Aaberge et al. (1998, 1999, 2001).

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[7] In Aaberge et al. (1998) the model is estimated on a sample containing only couples. The estimates for singles have been expressly produced for the present paper, and can be obtained upon request.

Appendix B

The dataset

The estimation and the simulation of the model is based on data from the 1993 Survey of Household Income and Wealth (SHIW93). This survey is conducted every two years by the Bank of Italy and besides household and individual socio-demographic characteristics, contains detailed information on labor, income and wealth of each household component.

The sample that we select contains 4827 individuals (2160 couples, 310 single females and 206 single males. Singles and couples with income from self-employment are excluded from the sample: this is due to the assumption that their decision process may be substantially different from wage-employees' and typically involves a permanent element of uncertainty.

We have restricted the ages of the individuals to be between 18 and 54 in order to minimize the inclusion in the sample of individuals who in principle are eligible for retirement, since the current version of the model does not take the retirement decision into account.

Due to the above selection rules, the estimates and the simulations should be interpreted as conditional upon the decisions not to be self-employed and not to retire.

The labor incomes measured by the survey are net of social security contributions and of taxes on personal income. Therefore, in order to compute gross incomes we have to apply the "inverse" tax code. In turn, the "direct" tax code has to be applied to every point in each household's choice set to compute disposable income associated to that point. Hourly wage rates are obtained by dividing gross annual wage income by observed hours.

Appendix C

The Italian 1993 tax rule

Here we summarise the main features of the personal income tax system in 1993. The unit of taxation is the individual. To the individual total taxable income, the following marginal tax rates are applied:

Income (1000 LIT)	Marginal tax rate (per cent)
Up to 7,200	10
7,200 - 14,400	22
14,400 – 30,000	27
30,000 – 60,000	34
60,000 – 150,000	41
150,000 – 300,000	46
Over 300,000	51

Some expenditures (such as medical or insurance) can be deducted from income before applying taxes. Child allowances and dependent spouse allowances – up to the amount of the gross tax – can be subtracted from the tax. Conditional on the number of household members and household total income the head of the household receives family benefits.

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